Information Percolation Driving Volatility

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Abstract

This paper provides a microfoundation for the persistence of volatility of aggregate stock market returns. In a centralized market, a large number of individuals possess dispersed information about future dividends. Information is processed, transmitted, and aggregated in two ways: through *word-of-mouth communication* and through the trading process. Both mechanisms operate simultaneously to generate persistent volatility. The resulting information flow drives both returns and volume. Volatility is mostly concentrated in the short-term asset, defined as the claim to short-maturity dividends. The pronounced heterogeneity in investors' information endowments induces patterns of trade consistent with empirical findings.

Keywords: volatility clustering, GARCH, dynamic equilibrium, overlapping generations, information percolation, word-of-mouth, noisy rational expectations, centralized markets.

JEL Classification. D51, D53, D82, D83, G11, G12.

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1 Introduction

A sizeable empirical literature documents the persistence of the volatility of returns on the aggregate stock market and the strong positive contemporaneous correlation between volatility and trading volume. ARCH/GARCH type models, pioneered by Engle (1982) and Bollerslev (1986), have become by now standard tools for the empirical analysis of volatility dynamics. But these tools do not address the question of what accounts for the persistence of volatility and its positive correlation with trading volume.

A rather sparse theoretical literature investigates this question. The predominant argument is that clustered arrival of news generates clustered volatility (in line with French and Roll (1986), which recognized that information flow is the determinant of volatility). For example, Campbell and Hentschel (1992) assume that large pieces of news about stock dividends tend to be followed by other large pieces of news, clearly generating clustered volatility. Cao, Coval, and Hirshleifer (2002) show how frictions into the trading process can result in clustered arrival of news into prices. McQueen and Vorkink (2004) assume that the sensitivity of a representative agent to news is clustered, while Brock and LeBaron (1996) argue that volatility clustering is generated by slowly adapting beliefs. Finally, in Veronesi (1999), agent's learning generates time-varying uncertainty and thus time-varying volatility.

I contribute to this literature by building a theoretical model of persistent volatility. With respect to the above studies, the present model does not assume any initial persistence or any trading frictions; the persistence of volatility arises endogenously from the rich information structure of the model. Moreover, by featuring a multi-agent economy, the present model explains the positive contemporaneous correlation between trading volume and volatility. Similar with the above studies, I argue that information flow is indeed the main driver of volatility and that agents' learning is crucial in generating persistent volatility. Yet, agents' learning in the present model features a natural ingredient which is not included so far in theoretical models of trading.

The novelty is that agents transmit information through word-of-mouth communication (they use a *private* channel of learning). They do so at bilateral random meetings, as in the models of information percolation developed by Duffie and Manso (2007). This private channel of learning is embedded in an otherwise standard noisy rational expectations economy (Grossman and Stiglitz, 1980), in which agents are able to extract information from prices (they use a *public* channel of learning). In other words, in the present model talking is decentralized while trading is centralized. I show how these two channels of learning interact with each other, generating persistent volatility and a positive contemporaneous correlation between volatility and trading volume.

Mounting evidence suggests that a natural channel of information transmission—the direct interpersonal communication among investors—can play an important role in generating asset-price volatility and can explain observed patterns of trade. For instance, Shiller (2000, p. 155) writes "word-of-mouth transmission of ideas appears to be an important contributor to day-to-day or hour-to-hour stock market fluctuations," and Stein (2008, p. 2150) describes conversations as being "a central part of economic life." Shiller and Pound (1989) provide evidence that direct interpersonal communications are an important determinant of investors decisions. Shiller (2000) devotes an entire chapter to the subject of word-of-mouth communication and its possible effects in financial markets. Cohen, Frazzini, and Malloy (2008) and Hong, Kubik, and Stein (2005) document patterns of trade that can be interpreted as evidence of word-of-mouth communication, while Hong, Kubik, and Stein (2004) show that stock-market participation is influenced by social interaction.[1]

Duffie and Manso (2007) have developed the information percolation theory to study the transmission of ideas in decentralized markets, where agents meet randomly to trade. Instead, I assume that agents meet randomly to communicate, but trade in centralized markets. Theoretical models of trading in centralized markets *à la* Grossman and Stiglitz (1980) usually assume that agents do not talk about the information they possess, thus abstracting from an important aspect of economic life. It is thus a natural step forward to analyze, at least theoretically, the impact of word-of-mouth communication in a centralized markets setting.

Consider an economy populated with a continuum of investors. Each investor is endowed with private signals about future fundamentals. Investors meet randomly with their peers and talk. The meetings of a particular agent with other agents occur at a sequence of Poisson arrival times. During such meetings they exchange views on future fundamentals. More precisely, agents exchange their conditional distributions of future fundamentals. Trading takes place in centralized markets. Through the trading process, the asset price aggregates the private information held by individual investors. Unobserved supply shocks prevent average private signals from being fully revealed by the price. The private information flow elaborated through random meetings and the public information flow aggregated through prices give rise to a positive contemporaneous correlation between trading volume and volatility, consistent with empirical evidence. The main finding is that transitory moments of intense word-of-mouth communication generate spikes in volatility, followed by persistent descents, as we usually observe in financial data.

To understand the interdependence between the private and the public channels

[1]For further evidence of social interaction in financial markets, the reader can refer to Grinblatt and Keloharju (2001), Feng and Seasholes (2004), Brown, Ivkovic, Smith, and Weisbenner (2008), Ivkovic and Weisbenner (2005), Massa and Simonov (2011), and Shive (2010).

of learning, consider two extreme cases: the *perfect information economy*, in which agents have perfect information about future fundamentals, and the opposite case, i.e., the *no information economy*. In both cases, prices and word-of-mouth communication play no role in aggregating knowledge—in the first case agents already have perfect information, whereas in the second case there is no information to be aggregated. Consider now the intermediate case, in which agents have differential information about future fundamentals—the *heterogeneous information economy*. In this case, the equilibrium disagreement across investors first increases and then decreases in signal noise. As a result, prices convey information. They help investors to revise their estimates of other agents' private signals. Additionally, once word-of-mouth communication takes place, investors know that the price is a better aggregator of private information. When forming their expectations about future dividends, they rely more on the price. Since the price is driven by fundamental shocks and supply shocks, the overreliance on prices magnifies the impact of these shocks. This increases the volatility of asset returns.

Once the volatility increases, it slowly descends. This result arises naturally if I assume that the intensity of word-of-mouth communication is time-varying. In recent empirical contributions, Da, Engelberg, and Gao (2011) and Vlastakis and Markellos (2010) use Google search frequency to capture an index of attention for retail investors. These studies clearly show that the willingness of agents to search for information is not constant. Motivated by this fact, I assume that the *meeting intensity* among agents follows a Markov Chain process with two states. Using insights from the aforementioned studies, I calibrate the process on Google search frequency data, and show that a transitory period of intense word-of-mouth communication produces a long-lasting effect on the volatility. After a period of intense word-of-mouth communication, agents gather on average a large amount of private signals. This large amount of private signals is exchanged at future meetings and perpetuates the high sensitivity of the price to fundamental and supply shocks. Consequently, the volatility becomes persistent. The persistence arises although the shock on the meeting intensity might be only transitory.

Moreover, fluctuations in the meeting intensity induce persistent effects in the dynamics of trading volume, for two reasons. First, since investors possess on average a large amount of signals once the word-of-mouth communication intensifies, they trade more aggressively. Second, a higher meeting intensity might increases disagreement across investors and force them to use price movements as information on which to make trading decisions. Hence, trading volume will be amplified by large price movements. Consequently, the trading volume increases and becomes positively related with the volatility. As in Andersen (1996) and Bollerslev and Jubinski (1999), the information flow creates a long-run dependency between trading volume and volatility.

Two additional implications arise from the model. The first is related to the recent finding of Binsbergen, Brandt, and Koijen (2010) that a large amount of the volatility is concentrated in the short-term asset, defined as the claim to short-maturity dividends. Leading asset pricing models generally predict the opposite, and thus are challenged by this finding. Consistent with Binsbergen et al. (2010), I show that information percolation increases the volatility mostly in the short-term. Intuitively, disagreement depends on the maturity of dividends, and thus the resulting term structure of disagreement dictates a term structure of volatility. The second implication is related to the empirical finding of Brennan and Cao (1997). Their paper shows within an international finance setup that better informed investors (i.e., domestic investors) act as contrarians, whereas poorly informed investors (i.e., foreign investors) act as trend-followers. In the current model, as agents start meeting with each other, they become heterogeneous with respect to their information endowment. This endogenously generated asymmetry in information endowments leads to different investment strategies: agents who have been efficient at gathering signals tend to act as contrarians, whereas agents who collected only a few signals tend to act as trend followers, confirming the evidence from Brennan and Cao (1997).

2 Related Literature

The modeling approach integrates two strands of literature. First, it has in common with the literature on noisy rational expectations that asset prices aggregate the private information held by individual agents and become public signals. Dynamic models from this literature typically assume that investors have private information about one-period-ahead fundamentals. This makes them only partially suited for my goal; the reason, as I will show, is that word-of-mouth communication has an impact on prices only if information is long-lived. The two exceptions are Bacchetta and Wincoop (2006) and Albuquerque and Miao (2010), who assume long-lived or "advanced" information. My model is closely related with these papers. Bacchetta and Wincoop (2006) offer a possible rationale for the disconnect between exchange rates and observed fundamentals, but abstract from word-of-mouth communication and its effect on stock market volatility, which is the focus of the present study. Albuquerque and Miao (2010) build a model of asymmetric information to explain short-run momentum and long-run reversal. In contrast with Albuquerque and Miao (2010), my model considers differential information. This crucial difference ensures that the signals exchanged by investors at random meetings are different and makes the information percolation channel relevant.

Second, it has in common with the literature on information percolation that the private information of individual investors is transmitted through the market by random meetings between them. Duffie and Manso (2007) borrow the term "percolation" from physics and chemistry, where it concerns the movement and filtering of fluids through porous materials. In economics, it concerns the dissemination of information of common interest through large markets. While Duffie and Manso (2007) focus on a decentralized market setting, Andrei and Cujean (2011) show that the percolation of information is particularly suitable for centralized markets models with dispersed information. The present paper follows the latter approach. Instead of assuming that agents meet randomly to trade, I let agents trade in a centralized market and meet randomly only to gather information. That is, markets are centralized, but information is not.

Most related papers trying to explain the persistence of volatility are Peng and Xiong (2002) and McQueen and Vorkink (2004). The model of Peng and Xiong (2002), building on Bookstaber and Pomerantz (1989), illustrates how the arrival of news in stock prices is clustered, even though the generation of news is i.i.d. The result arises because financial analysts digest news at a rate that endogenously changes through time. I echo the views expressed in Peng and Xiong (2002) that market takes time to digest information, generating persistent volatility. In the model of Peng and Xiong (2002), however, the price is related in a simple and mechanical way to news, while the current paper provides an equilibrium justification for the price. McQueen and Vorkink (2004) develop a preference model where investors' attitude toward risk and the attention they pay to news are affected by wealth shocks. This generates variations in their sensitivity to information. Although the behavioral model of McQueen and Vorkink (2004) offers valuable insights in explaining the asymmetry in the volatility, their assumption of persistent sensitivity to news is crucial in generating persistent volatility. In the present setup no variable is exogenously persistent, yet the volatility is. Finally, neither of these two papers bears any implication on trading volume and its link with the volatility, or emphasize the impact of disagreement and of the informational role of prices, which makes the current paper complementary to both of them.

3 A Dynamic Model of Word-of-Mouth Communication in Centralized Markets

The building blocks of the model are dispersed private information and word-of-mouth communication among investors. This additional channel of information transmission endogenously generates a very particular information structure, that I shall describe in this section.

3.1 Setup

The economy is populated by a continuum of rational agents, indexed by i, with CARA utilities and common risk aversion parameter γ. The agents consume a single good and live for two periods, while the economy goes on forever. Agent i in generation t is born with wealth w_t^i, and consumes wealth w_{t+1}^i in the next period. There is one risky asset (stock) and a riskless bond assumed to have an infinitely elastic supply at positive constant gross interest rate R. Both securities pay in units of the consumption good. At the beginning of period t, the stock pays a stochastic dividend D_t per share. D_t follows the process:

$$D_t = (1-\kappa_d)\,\bar{D} + \kappa_d D_{t-1} + \varepsilon_t^d, \qquad 0 \leq \kappa_d \leq 1. \tag{1}$$

The dividend innovation ε_t^d is i.i.d. with normal distribution $\varepsilon_t^d \sim \mathcal{N}(0, \sigma_d^2)$.

Per capita supply of the stock X_t is stochastic and follows the process:

$$X_t = (1-\kappa_x)\,\bar{X} + \kappa_x X_{t-1} + \varepsilon_t^x, \qquad 0 \leq \kappa_x \leq 1. \tag{2}$$

The dividend innovation ε_t^x is i.i.d. with normal distribution $\varepsilon_t^x \sim \mathcal{N}(0, \sigma_x^2)$. The noisy supply prevents the equilibrium asset price from completely revealing the average of the private information and thus ensures the existence of an equilibrium.

The common risk aversion assumption ensures that there is no trade motive due to differences in risk aversion (Campbell, Grossman, and Wang, 1993). Instead, investors trade only to accomodate noisy supply or to speculate on their private information. Dynamic noisy rational expectations models with similar structures are, for example, Watanabe (2008), Bacchetta and Wincoop (2006), and Banerjee (2010). Although the dividend and supply processes are quite general in the present setting, the results obtain already within an i.i.d setup. When i.i.d., the effect of the information percolation is completely isolated from other persistence effects.

The assumption of overlapping generations simplifies the analysis significantly, because it rules out dynamic hedging demands.[2] In Appendix A.7, I compute the solution of the model with infinitely lived agents, and show that results are almost identical with the overlapping generations case, although the numerical procedure is severely complicated. A similar result has been found by Bacchetta and Wincoop (2006) and Albuquerque and Miao (2010).

Investors allocate optimally their wealth between the risky stock and the safe asset. Let P_t be the ex-dividend share price. Each investor choses the holding of the risky

[2]Other papers adopt this assumption for tractability, such as Biais, Bossaerts, and Spatt (2003), Bacchetta and Wincoop (2006), Allen, Morris, and Shin (2006), Watanabe (2008), Bacchetta and Wincoop (2008), Albuquerque and Miao (2010), and Banerjee (2010).

asset x_t^i to maximize

$$\mathbb{E}_t^i\left(-e^{-\gamma \tilde{w}_{t+1}^i}\right) \tag{3}$$

subject to

$$\tilde{w}_{t+1}^i = (w_t^i - x_t^i P_t)R + x_t^i\left(P_{t+1} + D_{t+1}\right).$$

As is customary in the rational expectations literature, the price is conjectured to take a linear form of the model innovations. Consequently, the normality assumption and the CARA utility lead to the standard asset demand equation

$$x_t^i = \frac{\mathbb{E}_t^i\left(P_{t+1} + D_{t+1}\right) - RP_t}{\gamma \mathrm{Var}_t^i\left(P_{t+1} + D_{t+1}\right)}. \tag{4}$$

The market equilibrium condition is

$$\int_i x_t^i di = X_t. \tag{5}$$

This market clearing condition provides the equilibrium asset price that is a time-invariant linear function of innovations, as it will be described in Section 3.4.

3.2 Information Structure

All investors observe the past and current realizations of dividends and of the stock prices. Additionally, each investor observe an informational signal about the dividend innovation $T > 1$ periods later:

$$v_t^i = \varepsilon_{t+T}^d + \varepsilon_t^{vi}.$$

The private signal innovation ε_t^{vi} is i.i.d. with normal distribution $\varepsilon_t^{vi} \sim \mathcal{N}\left(0, \sigma_v^2\right)$.

Under this form, the present setup differs from existing models in several ways. Unlike in Watanabe (2008), where investors observe a signal about one-period-ahead dividends, in the present setup the signal is informative about further-away dividends; this makes the information percolation relevant, as agents have now several opportunities to trade until the dividend is actually realized. In a similar setup, Albuquerque and Miao (2010) name this signal "advance information". However, in the present case the information is dispersed, while in Albuquerque and Miao (2010) the private signal is common for all the informed investors. Having an infinity of heterogeneous private signals is crucial in this setup, otherwise agents have no reason to communicate at their random meetings.

The present modeling approach builds on Bacchetta and Wincoop (2006, 2008), who show that the "advance information" coupled with heterogeneous private signals generates higher order beliefs of a dynamic nature, which in turn disconnect the price from its fundamental value. Although these studies provide valuable insights about price dynamics, they abstract from word-of-mouth effects and its implications for the dynamics of volatility, which is the primary focus of the present work.

To isolate the effect of the information percolation, I assume that investors do not receive any new of private signals regarding the dividend innovation ε^d_{t+T} in the subsequent periods, i.e., from $t+1$ to $t+T-1$. A key feature provided by the information percolation is that, even if investors are endowed with private information only once, the random meetings between them—to be described shortly—are equivalent to bringing new information, and give them reasons to trade for informational purposes. Without word-of-mouth effects, investors would trade only for market making purposes, in order to accommodate the noisy supply.

3.3 Information Percolation

The Austrian economist F.A. Hayek was the first to realize that knowledge is not given to anyone in its totality: "the data are never for the whole society given to a single mind" (Hayek, 1945, p. 519). Instead, knowledge is dispersed throughout the marketplace. Hayek came to understand that the price mechanism aggregates knowledge residing within the market, and becomes a good indicator of everyone else's information.

Hayek considered the price mechanism very important, and put it on the same level as language. One should not neglect, though, the information processing power of language—particularly the communication of information from one person to another; a long time ago this innate channel of processing information was virtually the only form of information transmission and still has a powerful impact on human behavior.

In the context of financial markets, a relevant example happened in 1995, when IBM secretary Lorraine Cassano was asked to photocopy some papers. These papers included references to a top-secret takeover of software giant Lotus by IBM. Though forbidden from telling anyone about the takeover, she told her husband. The illicit information ultimately was passed down to six tiers of traders—a network of relatives, friends, co-workers, and business associates. After only 6 hours of word-of-mouth communication, the information reached twenty-five individuals; illegal trading generated profits of more than $1.3 million.[3]

The contributions of Grossman (1976), Diamond and Verrecchia (1981), and Hellwig

[3]Source: U.S. Securities and Exchange Commission Litigation Release No. 16161, MAY 26, 1999. Securities and Exchange Commission v. Lorraine K. Cassano, et al., Civil Action No. 99-CV -3822 (S.D.N.Y.)

(1980) formalized mathematically the thoughts of Hayek with respect to the price mechanism, while matters with respect to interpersonal communication are the focus of a separate literature. In economics, for instance, many theoretical models are based on the mathematical theory of the spread of disease. News propagate from "contagious" people (i.e., informed individuals) to "susceptible" people (i.e., uninformed individuals). The propagation takes place at a given *infection rate*, while people become no longer contagious at a given *removal rate*. In a recent contribution, Burnside, Eichenbaum, and Rebelo (2010) explain with such a model the moves on the housing market.[4]

A second and more recent approach to predict the course of word-of-mouth transmission of ideas in financial markets, denoted information percolation, has been developed by Duffie and Manso (2007). In this approach, people meet randomly with each other and exchange information. That is, instead of being only "contagious" or "susceptible", the type of individuals is defined by the amount of signals gathered; a richer structure than in epidemic models. Moreover, in Duffie and Manso (2007) the information flows in both directions, avoiding the sender-receiver dichotomy of epidemic models.

Duffie and Manso (2007) focus on a decentralized market setting, in which investors meet randomly to trade. Andrei and Cujean (2011) show that the percolation of information can be embedded in centralized markets models with dispersed information, formalizing the whole idea of Hayek—that both the price formation and the language contribute to the aggregation of knowledge. In Andrei and Cujean (2011) investors meet randomly to talk but trade in centralized markets. The present approach generalizes the setup from Andrei and Cujean (2011) to a dynamic setting, with the aim to show the impact of information percolation on the dynamics of the volatility.

In the context of the model that I formalized so far, let us focus for simplicity on $T = 3$ from now on. A general model can be considered at the expense of greater numerical computation. $T = 3$ represents the minimum time span necessary to show the effect of the information percolation on the persistence of the volatility and to seize the effect of the increasing precision as dividend materialize. Moreover, as it will be shown below, a graphical representation of the probability density function over the number of signals collected by each investor is still possible with $T = 3$.

This section shows how the initial signals received at date t become more and more relevant as the economy approaches date $t+3$. As already mentioned, at time t investors only receive a single private signal about the dividend innovation ε^d_{t+3}. At times $t+1$ and $t+2$, investors receive additional signals about ε^d_{t+3}, yet in a very specific way to be explained in this section. From date t onward, the agents meet with each other and

[4] Other related examples are Carroll (2006) and Hong, Hong, and Ungureanu (2010).

share truthfully the private signals that they received at date t. Any particular agent is matched to other agents at a sequence of Poisson arrival times with mean arrival rate λ, which is constant and common across agents.

Stock markets and the economy in general are often the subject of endless discussions among individuals. The reason is that every investor perceive these as important topics, as they represent opportunities or threats to personal wealth. For this reason, I abstract in the present setup from issues of strategic communication and just assume that, when two agents meet, they communicate truthfully their information. Moreover, since this economy can be envisioned as an ocean of agents, it will be difficult to find a strategic reason to lie. Similarly, Hong et al. (2010) consider that investors are "friends" and when they meet the informed investor tells the truth to the uniformed one. Likewise, Duffie, Malamud, and Manso (2009) do not model any incentive for matched agents to share information.[5]

To grasp the intuition, let us do the reasoning step by step, starting at time $t-2$, when agents receive signals about ε^d_{t+1}. These signals are still be valuable at time t, since they are informative about D_{t+1}. Between $t-2$ and $t-1$, investors meet with their peers during one period and exchange their signals about ε^d_{t+1}. At time $t-1$, before dying, investor i passes on her private information to the next investor i born the following period. Additionally, at time $t-1$ investors receive new signals about ε^d_{t+2}, still valuable at time t. Between $t-1$ and t, the investors continue meeting with each other and exchanging signals both about ε^d_{t+1} and ε^d_{t+2}. Therefore, at time t, each investor $i \in [0,1]$ is endowed with a random number $n^i_{t,1}$ of signals about ε^d_{t+1} (that is, for the dividend occurring one-period-ahead), with $n^i_{t,2}$ signals about ε^d_{t+2} (that is, for the dividend occurring two-periods-ahead), and with one signal about ε^d_{t+3}, as shown in Figure 1.[6]

By Gaussian theory, it is enough for the purpose of updating the agents' conditional expectation that agent i tells his counterparty at any meeting at time t his current conditional mean of the private signals and the corresponding total number of signals that he has gathered up to time t.

[5] Alternatively, as in Demarzo, Vayanos, and Zwiebel (2003), it can be assumed that agents consider information that they receive having a lower precision. That is, they might assign a lower precision to other agents. In this case, strong information percolation will wipe out the added noise and thus produce similar results. I have avoided this alternative for simplicity.

[6] One might be concerned that, doing so, some artificial correlations arises among signals. Although this overlapping feature could be interesting in its own way (see, e.g., Demarzo et al., 2003), it is not present in this model. At each point in time, agents share their initial signal and the signals that they have gathered by means of word-of-mouth communications. Since overlapping meeting between agents are ruled out (with a continuum of agents, the probability to meet again is zero), the signals gathered from a meeting are always different. Moreover, at each trading session, private signals are wiped out through aggregation. Consequently, agents consider signals accruing from further meetings as being genuinely new.

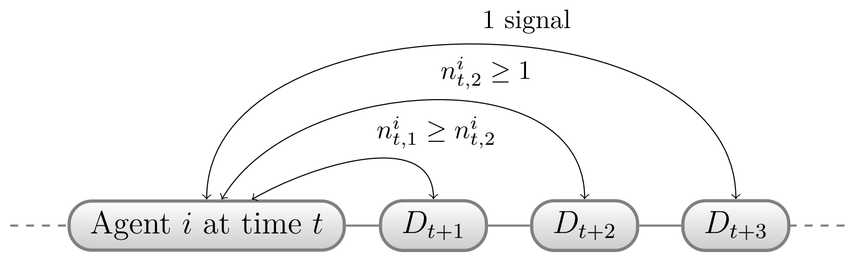

Figure 1: Information Percolation.

At time t, agent i is endowed with one private signal about ε^d_{t+3}, $n^i_{t,2} \geq 1$ private signals about ε^d_{t+2}, and $n^i_{t,1} \geq n^i_{t,2}$ private signals about ε^d_{t+1}.

The pair $\{n^i_{t,1}, n^i_{t,2}\}$ follows a probability density function on the support $\mathbb{N}^* \times \mathbb{N}^*$. The aim of this section is to compute this probability density function in closed form. It is straightforward to show that, for any investor $i \in [0, 1]$, we have $1 \leq n^i_{t,2} \leq n^i_{t,1}$, $\forall i$. If $\lambda > 0$, then $1 \leq n^i_{t,2}$. Since at time $t-1$ all agents start with at least one signal about ε^d_{t+1} and only one signal about ε^d_{t+2}, after one period of meetings (between $t-1$ and t) no investor can end up with more signals about ε^d_{t+1} than about ε^d_{t+2}. It follows that $n^i_{t,2} \leq n^i_{t,1}$.

The above statement implies that the marginal distribution of the number of signals at time t about ε^d_{t+1} and ε^d_{t+2} are dependent. Intuitively, an agent who at time t has gathered a considerable amount of signals about ε^d_{t+2} will have at least as many signals about ε^d_{t+1}. Another implication is that, if $\lambda > 0$, the average agent will be better informed about the dividends that will materialize in the immediate future. For example, as time t approaches, investors will have on average more signals about ε^d_{t+1} than for ε^d_{t+2}, and more signals about ε^d_{t+2} than for ε^d_{t+3}. The reasoning can easily be extended for $T > 3$.

Sharing information is additive in number of signals. That is, whenever two agents of types $\{n^i_{t,1}, n^i_{t,2}\}$ and $\{n^j_{t,1}, n^j_{t,2}\}$ meet, both become of type $\{n^i_{t,1} + n^j_{t,1}, n^i_{t,2} + n^j_{t,2}\}$. Following Duffie et al. (2009), the Stosszahlansatz (Boltzmann) equation for the cross-sectional distribution μ_t of types is

$$\frac{d}{dt}\mu_t = \lambda\mu_t * \mu_t - \lambda\mu_t \tag{6}$$

The first term on the right hand side of (6) represents the gross rate at which new agents of a given type are created. The second term of (6) captures the rate of replacement of agents of a given type with those of some new type that is obtained through matching and information sharing.

By direct recursive computation of the convolution formula (6), the probability

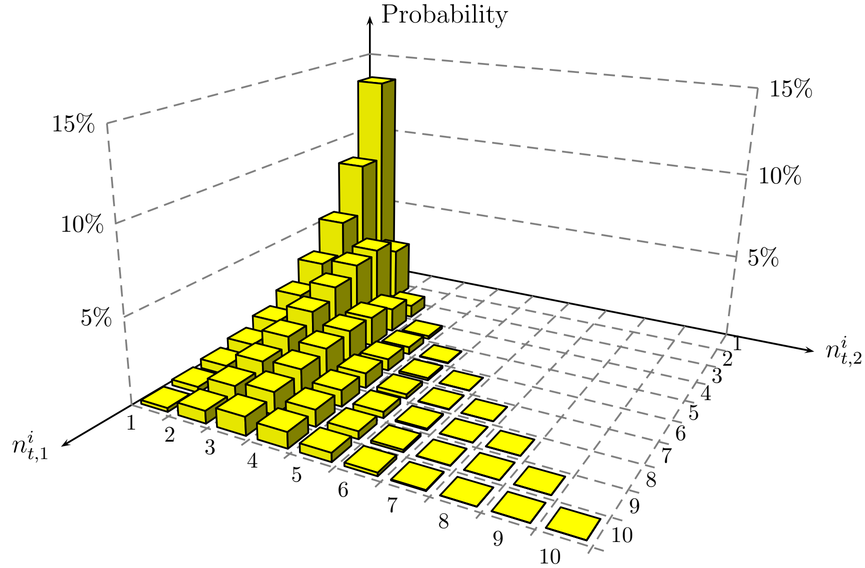

Figure 2: Distribution of the Number of Signals.
The probability distribution function of $\{n^i_{t,1}, n^i_{t,2}\}$, for $1 \leq n^i_{t,1} \leq 10$ and $1 \leq n^i_{t,2} \leq 10$. $n^i_{t,1}$ is represented on the left axis, while $n^i_{t,2}$ is represented on the right axis. For this example, the meeting intensity parameter is $\lambda = 1$.

distribution function of $\{n^i_{t,1}, n^i_{t,2}\}$ at time t, $\mu\left(n^i_{t,1}, n^i_{t,2}\right)$, is

$$\mu\left(n^i_{t,1}, n^i_{t,2}\right) = \begin{cases} \binom{n^i_{t,1}-1}{n^i_{t,2}-1} e^{-\lambda\left(n^i_{t,1}+n^i_{t,2}\right)} \left(e^{\lambda} - 1\right)^{n^i_{t,1}-1} & \text{if } n^i_{t,1} \geq n^i_{t,2}, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

A proof is provided in Appendix A.1.

With $T = 3$, the probability density function is bivariate and can still be represented graphically. Figure 2 shows the probability distribution function for a constant meeting intensity parameter $\lambda = 1$. As stated above, one can see that there is no probability mass whenever $n^i_{t,1} \geq n^i_{t,2}$. Further inspection of Figure 2 shows that, for $\lambda = 1$, approximately 10% of the agents still have $n^i_{t,1} = n^i_{t,2} = 1$ signal. These agents have not met any of their peers from $t-2$ to t. As an additional example, one can see on the same graph that approximately 3% of the agents have 4 signals about D_{t+1} and 2 signals about D_{t+2}.

Obviously, if $\lambda = 0$, then 100% of the population is of the type $n^i_{t,1} = n^i_{t,2} = 1$. In this case the investors are homogeneous with respect to the number of signals gathered and consequently with respect to their trading strategies. If λ increases substantially, the support of the distribution having non negligible probabilities may become very large. In that case, the heterogeneity of the investors with respect to the number of signals is very pronounced, and non negligible probabilities can be found even for large values of $n_{t,1}^i$ and $n_{t,2}^i$.

Finally, it is straightforward to show that the cross-sectional average of the number of signals $n_{t,1}^i$ is $\bar{n}_{t,1} = \exp(2\lambda)$, while the cross-sectional average of the number of signals $n_{t,2}^i$ is $\bar{n}_{t,2} = \exp(\lambda)$.

Word-of-mouth communication *endogenously* generates heterogeneity in information endowments. Although agents receive always one signal for the 3-periods ahead dividend (i.e., they start by being homogeneous with respect to the number of signals), they progressively become heterogeneous. This heterogeneity is a key implication of the model. Not only agents possess dispersed information about future dividends, but they also are asymmetrically informed. This information asymmetry generates heterogeneous trading strategies, as it will be shown in Section 6.

3.4 Learning

Having now established the setup and the information structure, a final prerequisite for the equilibrium computation is the characterization of the learning behavior of each agent i. The derivations mainly follow Bacchetta and Wincoop (2008). As standard, I consider solutions for the equilibrium price that are linear functions of model innovations. Thus, I conjecture the following equilibrium price:

$$P_t = \bar{\alpha}\bar{D} + \alpha D_t + \bar{\beta}\bar{X} + \beta X_{t-3} + (a_3\ a_2\ a_1)\epsilon_t^d + (b_3\ b_2\ b_1)\epsilon_t^x, \tag{8}$$

where $\epsilon_t^d \equiv (\varepsilon_{t+1}^d\ \varepsilon_{t+2}^d\ \varepsilon_{t+3}^d)^\top$ are the 3 future unobservable dividend innovations and $\epsilon_t^x \equiv (\varepsilon_{t-2}^x\ \varepsilon_{t-1}^x\ \varepsilon_t^x)^\top$ are the last 3 supply innovations.

The aim of this section is to compute in closed form $\mathbb{E}_t^i\epsilon_t^d$ and $\mathrm{Var}_t^i\epsilon_t^d$, that is, the individual conditional expectation and the individual conditional variance of the dividend innovations. I start first by collecting all the signals (public or private) of agent i. Then, by means of the projection theorem, I find the above mentioned conditional expectation and conditional variance.

In the present setup, the public signals are represented by prices. Define the vectors multiplying ϵ_t^d and ϵ_t^x in (8) a and b respectively. The adjusted price signal at time t, which contains only unobservable components at time t, is:

$$P_t^a = a\epsilon_t^d + b\epsilon_t^x.$$

Stated under the form (8), P_{t-1} and P_{t-2} contain information about future dividend innovations that is still useful at time t. Denote by $\mathbb{p}_t \equiv (P_t^a\ P_{t-1}^a\ P_{t-2}^a)^\top$ the set of price signals which contain only unobservable components at time t. This set can be written as

$$\mathbb{p}_t = \mathbb{A}\epsilon_t^d + \mathbb{B}\epsilon_t^x, \tag{9}$$

where $\mathbb{A}$ and $\mathbb{B}$ are 3×3 matrices composed of subsets of a and b, defined for convenience in Appendix Section A.2.

Turning now to private signals, each period t investor i obtain a single private signal about the dividend innovations at $t+3$:

$$v_t^i = \Omega\epsilon_t^d + \varepsilon_t^{vi},$$

where $\Omega \equiv [0\ 0\ 1]$. Furthermore, from the entire set of signals received from $t-2$ to t, a part is still valuable at time t—all the signals from the past that are informative about ε_{t+1}^d and ε_{t+2}^d, as Figures 1 and 2 suggest. More precisely, the investor i has $n_{t,1}^i$ signals about ε_{t+1}^d and $n_{t,2}^i$ signals about ε_{t+2}^d. By Gaussian theory, each of these signal sets is equivalent with a more precise single signal represented by the average within each set. Thus, the $n_{t,1}^i$ signals regarding ε_{t+1}^d are equivalent to one signal denoted hereafter by $w_{t,1}^i$, and the $n_{t,2}^i$ signals regarding ε_{t+2}^d are equivalent to one signal denoted hereafter by $w_{t,2}^i$. It follows that the past private signals can be grouped in

$$W_t^i \equiv \begin{pmatrix} w_{t,2}^i \\ w_{t,1}^i \end{pmatrix} = \Gamma\epsilon_t^d + \begin{pmatrix} \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix},$$

where $\Gamma \equiv \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ and $\varepsilon_{t,1}^{wi}$ and $\varepsilon_{t,2}^{wi}$ represent the innovations in the past private signals. The variance of these innovations must be adjusted to take into account the information percolation. With the number of signals gathered by agent i being $n_{t,1}^i$ and $n_{t,2}^i$, by Gaussian theory, the covariance matrix of $(\varepsilon_{t,2}^{wi}\ \varepsilon_{t,1}^{wi})$ is given by a 2×2 diagonal matrix whose diagonal elements are $\sigma_v^2/n_{t,2}^i$ and $\sigma_v^2/n_{t,1}^i$.

The vectors v_t^i and W_t^i jointly contain all current and past private signals available to agent i at time t that are informative about future dividend innovations. Given the assumption of uncorrelated errors in private signals, the averages of private signals across the population of agents are

$$\bar{v}_t = \Omega\epsilon_t^d \text{ and } \bar{W}_t = \Gamma\epsilon_t^d.$$

Grouping now all the available public and private information for investor i at time

t, the signals of future dividend innovations can be written as

$$\begin{pmatrix} \mathbb{p}_t \\ v_t^i \\ W_t^i \end{pmatrix} = \mathbb{H}\epsilon_t^d + \begin{pmatrix} \mathbb{B}\epsilon_t^x \\ \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix}, \tag{10}$$

where $\mathbb{H} \equiv (\mathbb{A}\ \Omega\ \Gamma)^\top$. Thus, each investor observe this 6-dimensional vector. The first part ($\mathbb{p}_t$) represents public information common to all investors, while the second part represents the individual private information of investor i. The variance of the errors in (10), that I shall denote by $\mathbb{R}^i$, is heterogeneous across investors because it depends on the number of signals that each investor gathered. Its computation is straightforward and is leaved for simplicity in Appendix A.2.

The errors in each of the signals from (10) have a normal distribution. The projection theorem implies that $\mathbb{E}_t^i\epsilon_t^d$ is given by the weighted average of these signals, with the weights determined by the precision of each signal. Appendix A.2 shows how direct application of the projection theorem leads to

$$\mathbb{E}_t^i\epsilon_t^d = \mathbb{M}^i \begin{pmatrix} \mathbb{p}_t \\ v_t^i \\ W_t^i \end{pmatrix} = \left(\text{Var}_t^i\epsilon_t^d\right)\mathbb{H}^\top\left(\mathbb{R}^i\right)^{-1} \begin{pmatrix} \mathbb{p}_t \\ v_t^i \\ W_t^i \end{pmatrix}, \tag{11}$$

with $\mathbb{M}_i = \sigma_d^2\mathbb{H}^\top\left(\sigma_d^2\mathbb{H}\mathbb{H}^\top + \mathbb{R}^i\right)^{-1}$. Furthermore, if one defines $\mathbb{I}_3$ as being the identity matrix of dimension 3, then

$$\text{Var}_t^i\epsilon_t^d = \sigma_d^2\left(\mathbb{I}_3 - \mathbb{M}^i\mathbb{H}\right) = \left(\frac{1}{\sigma_d^2}\mathbb{I}_3 + \mathbb{H}^\top\left(\mathbb{R}^i\right)^{-1}\mathbb{H}\right)^{-1}. \tag{12}$$

As I will show in the next section, $\mathbb{E}_t^i\epsilon_t^d$ and $\text{Var}_t^i\epsilon_t^d$ are sufficient for the equilibrium computation.

3.5 Equilibrium

Having now all the necessary results from the learning part, we can turn to the equilibrium price. The problem for an individual investor i, stated in (3), leads to the standard asset demand equation (4). Impose market clearing as in (5) to get

$$\frac{1}{\gamma}\left(\int_0^1 \frac{\mathbb{E}_t^i\left(P_{t+1} + D_{t+1}\right)}{\text{Var}_t^i\left(P_{t+1} + D_{t+1}\right)}di - RP_t\int_0^1 \frac{1}{\text{Var}_t^i\left(P_{t+1} + D_{t+1}\right)}di\right) = X_t \tag{13}$$

Equation (13) solves for the unknown coefficients $\bar{\alpha}$, α, $\bar{\beta}$, β, a_j, and b_j, for $j = 1, 2, 3$. For this, we first need $\mathbb{E}_t^i (P_{t+1} + D_{t+1})$ and $\text{Var}_t^i (P_{t+1} + D_{t+1})$. Then, the integrals in (13) are obtained from the conjectured price (8). Let us start first with $P_{t+1} + D_{t+1}$:

$$P_{t+1} + D_{t+1} = f(\bar{D}, D_t, \bar{X}, X_{t-3}) + a_1 \varepsilon_{t+4}^d + b_1 \varepsilon_{t+1}^x + a^* \epsilon_t^d + b^* \epsilon_t^x, \tag{14}$$

where $f(\cdot)$ is defined as a linear function of $\bar{D}$, D_t, $\bar{X}$, and X_{t-3}:

$$\begin{aligned} f(\bar{D}, D_t, \bar{X}, X_t) \equiv & \left[\bar{\alpha} + (\alpha + 1)(1 - \kappa_d)\right] \bar{D} + (\alpha + 1)\kappa_d D_t \\ & + \left[\bar{\beta} + \beta(1 - \kappa_x)\right] \bar{X} + \beta \kappa_x X_{t-3}, \end{aligned} \tag{15}$$

and $a^* = (\alpha + 1 \ a_3 \ a_2)$ and $b^* = (\beta \ b_3 \ b_2)$.

Equation (14) can be further simplified by using (9):

$$P_{t+1} + D_{t+1} = f(\bar{D}, D_t, \bar{X}, X_{t-3}) + a_1 \varepsilon_{t+4}^d + b_1 \varepsilon_{t+1}^x + \psi \epsilon_t^d + b^* \mathbb{B}^{-1} \mathbb{p}_t,$$

with $\psi = a^* - b^* \mathbb{B}^{-1} \mathbb{A}$. This gives the conditional variance of $P_{t+1} + D_{t+1}$:

$$\text{Var}_t^i (P_{t+1} + D_{t+1}) = a_1^2 \sigma_d^2 + b_1^2 \sigma_x^2 + \psi \left(\text{Var}_t^i \epsilon_t^d \right) \psi^\top.$$

By use of (9), the conditional expectation of $P_{t+1} + D_{t+1}$ becomes

$$\begin{aligned} \mathbb{E}_t^i (P_{t+1} + D_{t+1}) &= f(\bar{D}, D_t, \bar{X}, X_{t-3}) + a^* \mathbb{E}_t^i \epsilon_t^d + b^* \mathbb{E}_t^i \epsilon_t^x \\ &= f(\bar{D}, D_t, \bar{X}, X_{t-3}) + \psi \mathbb{E}_t^i \epsilon_t^d + b^* \mathbb{B}^{-1} \mathbb{A} \epsilon_t^d + b^* \epsilon_t^x. \end{aligned} \tag{16}$$

Equation (16) provides a hint on the informational role of prices. Because investors do not know whether price fluctuations are driven by supply shocks or by information about future dividends, the supply shocks enter in the individual conditional expectations. In the next section I show that precisely this feature—common to noisy rational expectations models and denoted *rational confusion* by Bacchetta and Wincoop (2006) or *informational effect* by Grundy and Kim (2002)—is responsible for the magnification of supply shocks on asset prices when word-of-mouth communication takes place.

We are able now to compute the integrals in (13). The details are leaved for simplicity in Appendix A.3. Proposition 1 defines the rational expectations equilibrium pertaining to this economy.

Proposition 1. ***(Equilibrium)*** *A rational expectations equilibrium for the described information structure is characterized by the following price function P_t, and the demand* *function x_t^i:*

$$P_t = \bar{\alpha}\bar{D} + \alpha D_t + \bar{\beta}\bar{X} + \beta X_{t-3} + a\epsilon_t^d + b\epsilon_t^x$$
$$x_t^i = \frac{\mathbb{E}_t^i \left(P_{t+1} + D_{t+1}\right) - RP_t}{\gamma \mathrm{Var}_t^i \left(P_{t+1} + D_{t+1}\right)},$$

with $\mathbb{E}_t^i \left(P_{t+1} + D_{t+1}\right)$ and $\mathrm{Var}_t^i \left(P_{t+1} + D_{t+1}\right)$ given by

$$\mathbb{E}_t^i \left(P_{t+1} + D_{t+1}\right) = f(\bar{D}, D_t, \bar{X}, X_{t-3}) + \psi \mathbb{E}_t^i \epsilon_t^d + b^* \mathbb{B}^{-1} \mathbb{A} \epsilon_t^d + b^* \epsilon_t^x$$
$$\mathrm{Var}_t^i \left(P_{t+1} + D_{t+1}\right) = a_1^2 \sigma_d^2 + b_1^2 \sigma_x^2 + \psi \left(\mathrm{Var}_t^i \epsilon_t^d\right) \psi^\top.$$

$\mathbb{E}_t^i \epsilon_t^d$, $\mathrm{Var}_t^i \epsilon_t^d$, and $f(\bar{D}, D_t, \bar{X}, X_{t-3})$ are given by (11), (12), and (15) respectively.

The coefficients $\bar{\alpha}$, α, $\bar{\beta}$, β, a_j, and b_j can be derived by solving a fixed point problem, equating the coefficients of the conjectured price to those in the market clearing condition. The system of nonlinear equations to be solved is provided in Appendix A.3.

Proof. See Appendix A.3.

In an infinite horizon model with overlapping generations, Spiegel (1998), Bacchetta and Wincoop (2008), Watanabe (2008), and Banerjee (2010) show that there exist multiple equilibria. The multiplicity is unrelated to information heterogeneity, but arises if the sources of risk in the model are too large. Intuitively, a too large current risk premium increases the risk premium in the previous period, which increases the risk premium in the period before, and so on. If the sources of risk in the model are too large, the risk premium might explode and thus an equilibrium might not exist.

In the present case, with only one risky security, there potentially exist 2 equilibria. The conditions for existence can be characterized only in special cases, since the system of equations needed to find the undetermined coefficients cannot be solved in closed form. Numerical result suggest, however, that there are two real roots corresponding to two stationary equilibria. As is customary found in the literature (see, e.g., Spiegel, 1998; Watanabe, 2008; Banerjee, 2010), there exist one high volatility equilibrium, and one low volatility equilibrium.

The low volatility equilibrium is the limit of the unique equilibrium in the finite version of the model (see Appendix A.4 or Banerjee (2010) for a proof). It is therefore a stable equilibrium. On the contrary, the high volatility equilibrium is unstable, because of the forward looking property of the volatility: it can be an equilibrium today if one believes that it will be the equilibrium at all future dates. Therefore, in the analysis that follows, I choose to focus on the low volatility equilibrium. Another reason for this choice is that, in order to explain the persistence of the volatility (in Section 4), a finite version of the model is needed.

3.6 Discussion of the Solution

The market clearing condition (13) defines the following form for the price:

$$P_t = \frac{1}{R}\frac{\int_0^1 \frac{\mathbb{E}_t^i(P_{t+1}+D_{t+1})}{\mathrm{Var}_t^i(P_{t+1}+D_{t+1})}di}{\bar{K}_t} - \frac{\gamma}{R\bar{K}_t}X_t = \frac{1}{R}\widehat{\mathbb{E}}_t\left[P_{t+1}+D_{t+1}\right] - \frac{\gamma}{R\bar{K}_t}X_t, \tag{17}$$

where $\bar{K}_t$ represents the average precision of the entire population of investors, defined in Appendix A.3. The equilibrium price has two terms. The first term (denoted henceforth P_t^*) is the weighted average of investors' expected future dividends, discounted at the risk free rate; the second term is the risk premium. Integrating (17) forward yields the stock price as the sum of the present discounted value of expected dividends (the fundamental value, Grundy and Kim, 2002) minus expected future and current risk premia that determine current and future discount rates.

The value P_t^* aggregates the expectations of all individual investors. These individual expectations, stated in equation (16), can be re-written as follows:

$$\mathbb{E}_t^i\left(P_{t+1}+D_{t+1}\right) = f\left(\bar{D}, D_t, \bar{X}, X_{t-3}\right) + \psi\mathbb{M}_2^i\begin{pmatrix} v_t^i \\ W_t^i \end{pmatrix} + \left(\psi\mathbb{M}_1^i + b^*\mathbb{B}^{-1}\right)\mathbb{p}_t \tag{18}$$

where $\mathbb{M}_1^i$ and $\mathbb{M}_2^i$ represent the first 3 and the last 3 columns of $\mathbb{M}^i$ respectively. Inspection of equation (18) shows clearly how each investor uses private signals (the second term) and prices (the third term) in forming her expectation. In representative agent economies, the last term of (18) is zero, while in the present model it reflects the informational role of prices. Through this term, investors use observed prices to learn about other investors' private information.

The following analysis uses equation (18) to quantify the combined effect of the information percolation and the informational role of prices on the coefficients of dividend and supply shocks.

Information percolation and dividend shocks

In the present structure of the model, the price depends on three dividend shocks, ε_{t+1}^d, ε_{t+2}^d, and ε_{t+3}^d. For the sake of brevity, I perform here the analysis related to the dividend shock ε_{t+1}^d. The results for the other two dividend shocks are qualitatively similar and thus bear the same interpretations.

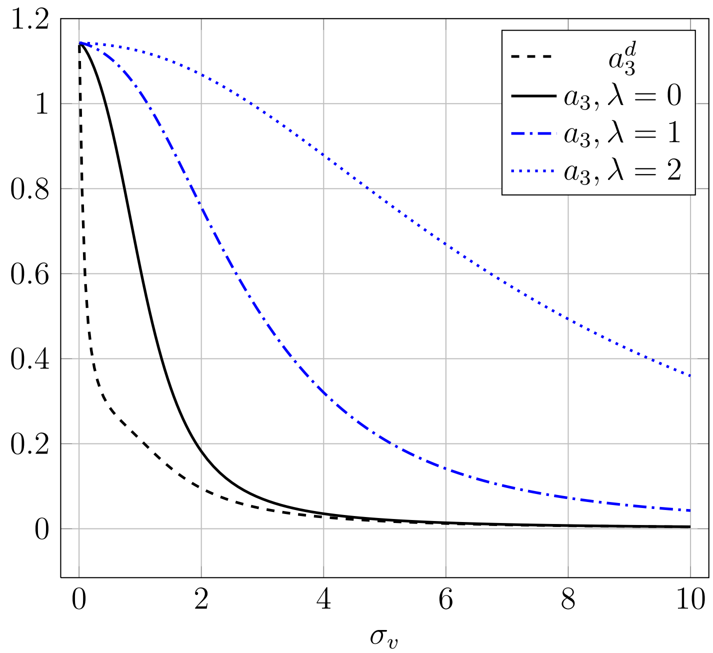

Figure 3: Coefficient of dividend shock ε_{t+1}^d.
The black solid line shows the coefficient a_3 in a standard noisy rational expectations without information percolation ($\lambda = 0$). The black dashed line shows the direct effect, arising if agents learn only from private signals, a_3^d. The blue lines show how information percolation modifies the coefficient a_3. The parameter values are calibrated to match the monthly returns and volatilities of the aggregate stock market (see discussion in Section 3.7): $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

The coefficient of the dividend shock ε_{t+1}^d in the equilibrium price is a_3. This coefficient can be split into two parts. The first part reflects the direct effect of the private information (the second term in equation 18), while the second part represents the effect produced by the informational role of prices (the third term in equation 18). Denote by a_3^d the first (direct) effect and by a_3^p the second effect (produced by prices). It follows that $a_3 = a_3^d + a_3^p$.

The black solid line in Figure 3 depicts the coefficient a_3 as a function of the private signal variance. The case considered is $\lambda = 0$, i.e., standard rational expectations without information percolation. When the variance of private signals tends to zero (the perfect information case), the coefficient a_3 is positive and reaches an upper bound. In this case, investors perfectly forecast the future dividends. In contrast, when the variance of private signals tends to infinity (the no information case), investors do not have any private information to rely on and thus the coefficient a_3 tends to zero. In this case, the price do not respond to any dividend shocks.

The black dashed line in Figure 3 depicts the direct effect, produced only by the private information channel, a_3^d. In the perfect information case ($\sigma_v \to 0$), the two lines meet. The reason is that, when information is perfect, there is no informational role of prices. In the no information case ($\sigma_v \to \infty$), the two lines meet again. The reason is that, when there is no private information, prices do not aggregate any information. For intermediate values of the variance of private signals the informational role of prices comes into play and increases the coefficient a_3. The distance between lines a_3^d and a_3 is nothing else than a_3^p. Thus, the informational role of prices amplifies the impact of dividend shocks on the price.

What happens when information percolation takes place? The blue lines (dot-dashed and dotted) in Figure 3 show that a_3 is sensibly magnified. This happens because prices are more informative when information percolation takes place. If the information percolation intensifies, agents rely more on prices to build their expectations of future fundamentals. This amplifies further away the impact of dividend shocks on the price.

Information percolation and supply shocks

In the present structure of the model, the price depends on three supply shocks, ε_{t-2}^x, ε_{t-1}^x, and ε_t^x. For the sake of brevity, I perform here the analysis related to the supply shock ε_t^x. The results for the other two supply shocks are qualitatively similar and thus bear the same interpretations.

The coefficient of the supply shock ε_t^x in the equilibrium price is b_1. As in the dividend shock case, this coefficient can be split in two parts. The primary effect arises directly through the risk-premium channel, as expressed in (17). The second effect is produced by the informational role of prices (the third term in equation 18). Denote by b_1^d the first (direct) effect and by b_1^p the second effect (produced by prices). It follows that $b_1 = b_1^d + b_1^p$.

The black solid line in Figure 4 depicts the coefficient b_1 as a function of the private signal variance, while the black dashed line depicts the coefficient b_1^d. Because prices play no informational role when $\sigma_v \to 0$ or $\sigma_v \to \infty$, the two lines meet in both cases. For intermediate values of the variance of private signals the informational role of prices magnifies the coefficient b_1, and thus amplifies the effect of supply shocks.

This magnification arises because supply shocks are unobservable, which makes price fluctuations only imperfect signals about future fundamentals. Too see this, start from equation (17) and compute price changes

$$\Delta P_t = \Delta P_t^* - \frac{\gamma}{R\bar{K}_t}\Delta X_t.$$

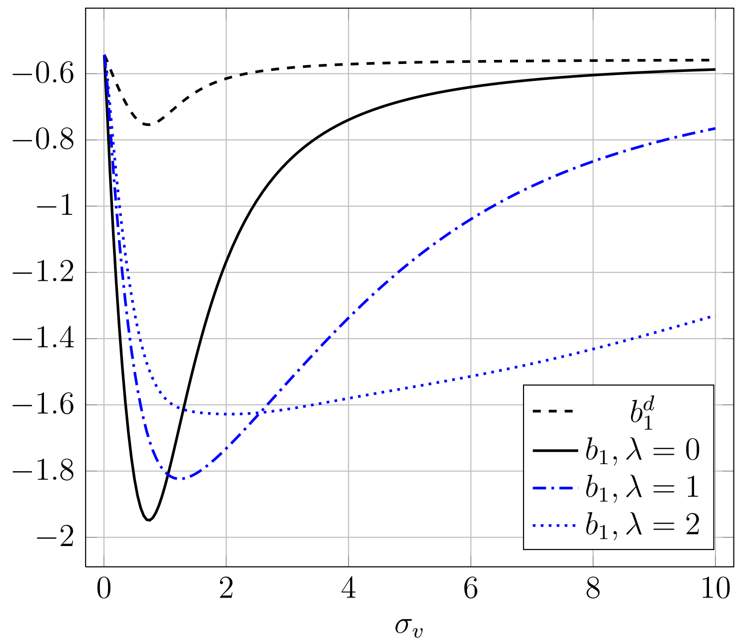

Figure 4: Coefficient of supply shock ε_t^x.
The black solid line shows the coefficient b_1 in a standard noisy rational expectations without information percolation ($\lambda = 0$). The black dashed line shows the direct effect, arising through the risk premium channel, b_1^d. The blue lines show how information percolation modifies the coefficient b_1. The parameter values are calibrated to match the monthly returns and volatilities of the aggregate stock market (see discussion in Section 3.7): $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

The unconditional variance of price changes is

$$\text{Var}\left(\Delta P_t\right) = \text{Var}\left(\Delta P_t^*\right) + \left(\frac{\gamma}{R\bar{K}_t}\right)^2 \text{Var}\left(\Delta X_t\right) - 2\frac{\gamma}{R\bar{K}_t}\text{Cov}\left(\Delta P_t^*, \Delta X_t\right) \tag{19}$$

The last term in (19) arises because investors use observed prices to learn about other investors' private information. In representative agent economies, this term is zero, while in the present model it always increases the price variance, because fundamental value differences (ΔP_t^*) and supply differences (ΔX_t) move in opposite directions. Intuitively, an unobservable positive supply change make investors infer from the consequent decrease in price that the fundamental value might be lower. Thus, investors revise downward their forecast of future dividends, generating a negative fundamental value change. The reverse happens for a negative supply change.

The blue lines (dot-dashed and dotted) in Figure 4 show that b_1 is sensibly magnified when information percolation takes place. If the information percolation intensifies, agents rely more on prices to build their expectations of future fundamentals. This amplifies further away the impact of supply shocks on the price.

The effect of the variance of private signals on the supply shock coefficient b_1 is non-monotonic. It arises because two opposite forces are at work. A higher variance of private signals pushes investors to increase the weight given to prices, for the private signals are less informative. This force enhances the informational role of prices and thus strengthens the coefficient b_1. However, a higher variance of private signals pushes investors to rely less on prices, for their informational role decreases. This force weakens the coefficient b_1. One of these two forces dominates, depending on the value of the variance of private signals.

In the information percolation case, the effect is less ambiguous than in a classic noisy rational expectations equilibrium. The solid black line shows that the coefficient b_1 becomes important only for a narrow range of values of the variance of private signals, whereas in the information percolation case the results are robust for a wider range of values. The information percolation restores the balance in the favor of the first force. That is, the information percolation induces over-reliance on public information.

The importance of the information percolation can be quantified by computing magnification factors. In the case of dividend shocks, the magnification factor is equal to a_3/a_3^d. It quantifies clearly how the direct effect arising from learning of private information only is multiplied once prices play their informational role. Panel (a) of Figure 5 depicts the magnification factor in a standard rational expectations (solid line) and when information percolation takes place ($\lambda = 1$, dashed line). One can see that the effect of dividend shocks is greatly amplified in an economy with information percolation: the magnification factor take values as large as 12. If $\lambda = 2$, separate calculations show that the magnification factor can be as high as 80.

In the case of supply shocks, the magnification factor is equal to b_1/b_1^d. It quantifies clearly how the direct effect arising through the risk premium channel is multiplied once prices play their informational role. Panel (b) of Figure 5 depicts the magnification factor in a standard rational expectations (solid line) and when information percolation takes place ($\lambda = 1$, dashed line). One can see that the effect of supply shocks is greatly amplified in an economy with information percolation.

To summarize, the information percolation modifies the way information is elaborated (through random meetings) and aggregated (through prices). As a result, the impact of supply shocks is magnified because agents do not exactly know the origin of price fluctuations, whereas the impact of dividend shocks is magnified because the agents use current and past prices to improve their estimate of future dividends. Ultimately, both effects arise because agents use prices to infer information, a common feature of rational expectations models. But, while in rational expectations models the effect arises only for a narrow range of the variance of private information, the information percolation

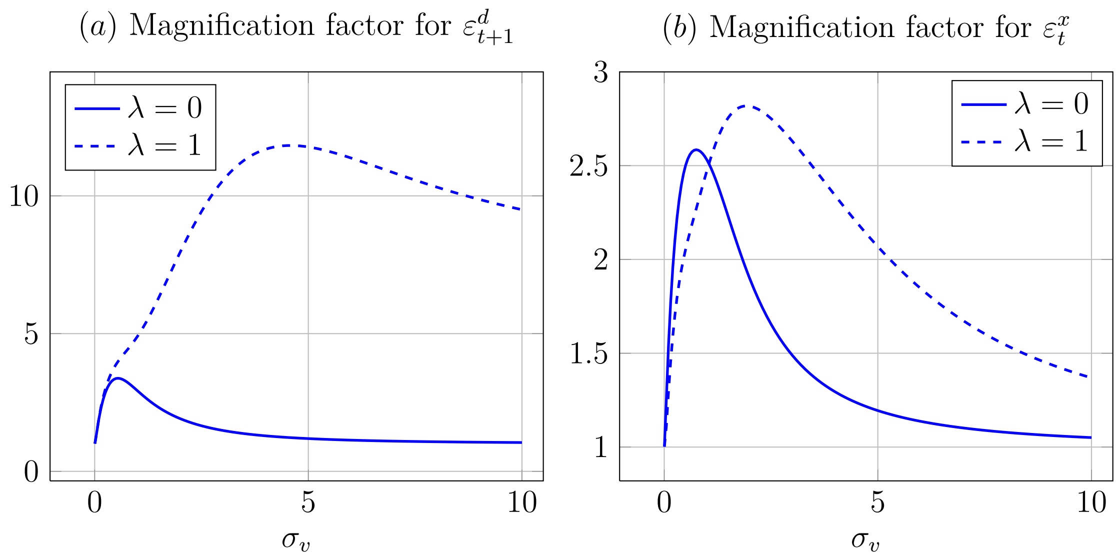

Figure 5: Magnification Factors.
The solid line depicts the magnification factor for different standard deviations of the private information error, without information percolation, i.e., $\lambda = 0$. The dashed line plots the magnification factor with information percolation, i.e., $\lambda = 1$. The parameter values are calibrated to match the monthly returns and volatilities of the aggregate stock market (see discussion in Section 3.7): $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

generates more powerful and robust results.

3.7 Benchmark Calibration

I use the calibration performed by Banerjee (2010) on stock market returns at monthly frequency. In his article, the parameter values are picked to match the monthly returns of the market portfolio over the period January 1983 to December 2008. The benchmark calibration is presented in Table 1.

The supply shocks are i.i.d. over time. First, this may seem reasonable at monthly frequency. The results are similar if supply shocks are assumed to be persistent. Moreover, since the aim is to explain the persistence of the volatility, it is preferable to eliminate any persistence that may arise from the supply part. The standard deviation of the private signals is assumed to be high with respect to the standard deviation of the dividends and of the supply shocks, as found by Cho and Krishnan (2000). A relatively large standard deviation of private signals is proposed as well in Bacchetta and Wincoop (2006) and Hassan and Mertens (2011). As shown in Section 3.6, though, the results hold for a wide range of values of σ_v. As an additional exercise, I performed the calculations using the calibration from Grundy and Kim (2002). The results are qualitatively similar.

Parameter	Symbol	Values
Risk aversion	γ	1
Gross interest rate	R	1.004
Long run mean of the supply	$\bar{X}$	0.176
AR(1) parameter noisy supply	κ_x	0
Long run mean of the dividends	$\bar{D}$	0.224
AR(1) parameter dividends	κ_d	0.129
Standard deviation of dividend shocks	σ_d	0.628
Standard deviation of supply shocks	σ_x	0.358
Standard deviation of private signal errors	σ_v	5

Table 1: Benchmark Calibration.

This calibration, inspired from Banerjee (2010), is picked to match the monthly returns of the market portfolio over the period January 1983 to December 2008.

4 Implications for the Volatility

The equilibrium price is a linear form of normally distributed variables. It is therefore normally distributed. It follows that price differences are normally distributed, which makes the computation of their variance easier. In the analysis that follows, I consider both price changes (dollar returns) and rates of returns. Since rates of returns are no longer normally distributed, I use simulations to compute their volatility. Both cases (dollar and rates of returns) are presented simultaneously. I report results for cum-dividend returns, as in Banerjee (2010), although the results for ex-dividend returns are stronger. The dollar excess returns are defined as

$$r^{\$}_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t, \tag{20}$$

whereas the rates of return are defined as

$$r_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t} - R.$$

Figure 6 shows the effect of the percolation on the volatility of asset returns. Both the dollar returns volatility and the rates or return volatility increase as the speed of information dissemination gets higher. Because investors communicate their information at a higher speed, the price becomes more sensitive to both dividend shocks and supply shocks, resulting in a higher return volatility.

4.1 Dynamics of the Volatility

The percolation concept is borrowed from physics with the aim to predict the course of word-of-mouth transmission of ideas. Nonetheless, a word of caution is needed. In

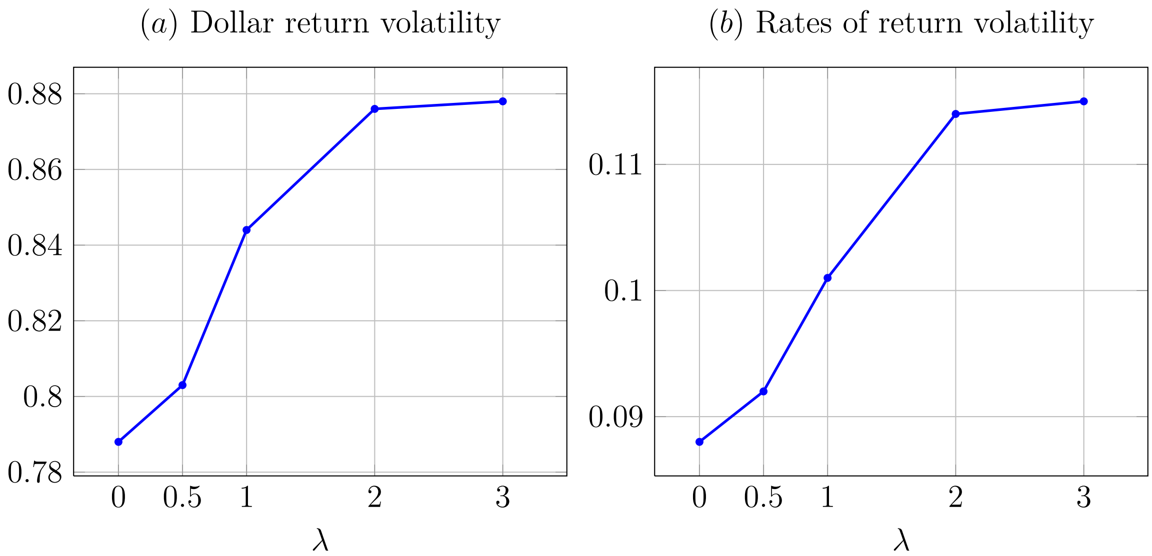

Figure 6: Volatility and Information Percolation.
Panel (a) depicts the dollar returns volatility in an economy with constant information percolation for different values of the parameter λ. Panel (b) depicts the rates of return volatility, annualized. The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

social sciences as opposed to physics, parameters are seldom constant. In the context of the information percolation model, the meeting intensity may suddenly increase, generating spread of *epidemic* news. For example, we often observe sudden spikes in the attention of the entire community on urgent economic matters, such as the "fiscal cliff," which was under intense discussion for weeks at the end of 2012. During such episodes, word-of-mouth communication can proceed with great speed across disparate social groups.

Two observations, both provided by Shiller (2000), are evidence on fluctuations of word-of-mouth communication intensity and their link with the volatility. The first is related with the survey study conducted by Shiller during the week of the stock market crash of 1987. Respondents reported that they talked intensively about the market situation during the day of the crash (individual investors talked on average to 7 other people, whereas institutional investors talked on average with 20 other people). The second observation is related to the obvious increase in the word-of-mouth communication intensity once the telephone become effective during the 1920s. The increased volatility of the stock market during that period, can be explained partially by the introduction of the telephone.

Adding to this evidence, two recent papers (Vlastakis and Markellos, 2010; Da et al., 2011) build a direct measure of investors' attention using Google search frequency data. As intuitively expected, some periods reveal the investor' strong willingness to gather

State		L	H	Unconditional Probability
$\lambda_t = 0.5$	L	0.90	0.79	0.88
$\lambda_t = 2$	H	0.10	0.21	0.12

Table 2: Information Percolation States.

There are two states, depending on the values of λ_t. The transition matrix Q of this two-states Markov chain is shown in columns 3-4. It controls the probability of a switch from state j (column j) to state i (row i). For example, this means that the probability of staying in state L is equal to 0.9. Likewise, the probability of a switch from state L to state H is 0.1. The sum of each column in this matrix is equal to one. The last column is obtained by computing the stationary distribution, i.e. $\lim_{n\to\infty} Q^n$.

information and some others don't. In other words, investors' incentive to acquire information varies through time. Moreover, Vlastakis and Markellos (2010) show that the attention index explains roughly 50% of the variability in the Market Volatility Index (VIX). Motivated by this evidence and by the observations of Shiller, I explore the effects of a time-varying meeting intensity on the volatility of asset returns.

For this, I start by assuming that the information percolation parameter follows a Markov Chain with 2 states. Periods of low meeting intensity (L) are alternating with periods of high meeting intensity (H). In the low meeting intensity state, I fix $\lambda_L = 0.5$, which broadly means that each agent meets one other agent every 2 months. In the high meeting intensity state, I fix $\lambda_H = 2$, which means that each agent meets on average 2 other agents per month.

The main result is that information percolation generates persistence in the volatility of asset returns. This arises even if the calibrated process for λ_t shows no persistency in the high meeting intensity state. Assume that the transition matrix for the Markov Chain process takes the values from Table 2. Let us abstract now from any empirical justification of these numbers. The next section will clarify this point.

If the economy is in a low meeting intensity state this month, then there is 90% chance that it will remain in a low meeting intensity state next month. On the contrary, if the economy is in a high meeting intensity state this month, then there is 21% chance that it will remain in a high meeting intensity state next month. The expected duration of the low meeting intensity state is approximately 100 days, whereas the expected duration of the high meeting intensity state is approximately 13 days. Unconditionally, the economy is in a low meeting intensity state in 88% of cases, and in a high meeting intensity state in 12% of cases.

Figure 7 illustrates the mechanism potentially amenable to produce persistent volatility. The dashed line, represented on the left axis, shows a two-years sample path of λ, simulated with the Markov chain parameters described above, at monthly frequency. One can see that a transitory shock has occurred at month 4, and a two-period lasting

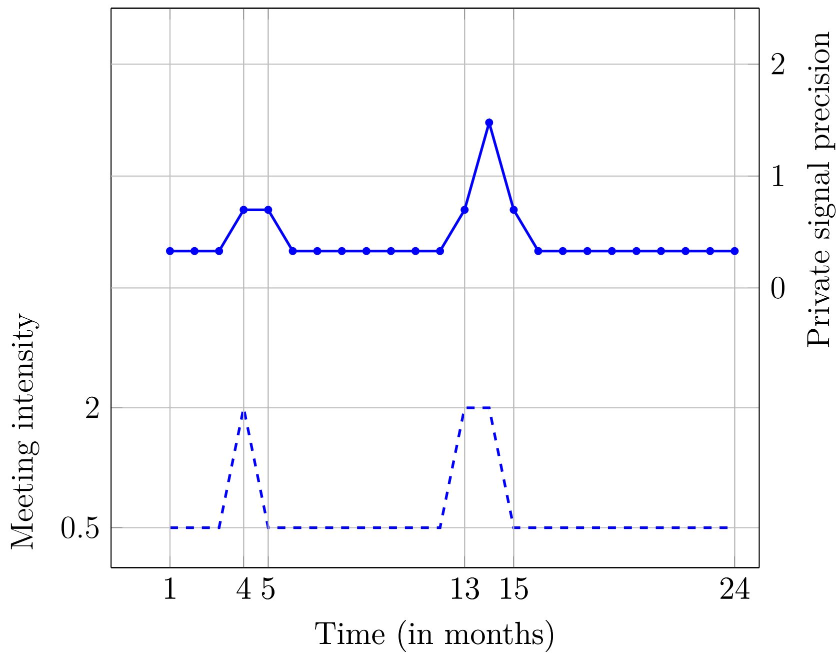

Figure 7: Average Precision Dynamics.
The dashed line shows a sample path of the meeting intensity λ_t, simulated with the Markov chain parameters from Section 3.7. The solid line shows the response of the average precision of the private signals about the nearest dividends, $\sqrt{\bar{n}_{t,1}}/\sigma_v$. The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

shock has occurred at month 13. The solid line, represented on the right axis, depicts the average precision of the private signals pertaining to the nearest dividend, $\sqrt{\bar{n}_{t,1}}/\sigma_v$. Following the shock at time 4, the average precision increases, since agents shared their private signals about D_5 at a higher speed during one period. In the meantime, the agents shared private signals about D_6 at a higher speed. Thus, even though at time 5 the transitory shock on the percolation vanishes, agents had already accumulated more signals about D_6 and therefore the average precision stays high for one more month. The same logic applies for the shocks from times 12 and 13. This reasoning is done for $T = 3$ but can be extended easily to $T > 3$, generating even more persistent shocks.

A long-lasting precision effect, as in Figure 7, translates in a long-lasting volatility effect. To show this, I compute the impact of the time-varying information percolation speed on the volatility of asset returns. This can only be measured in a finite version of the model. The infinite horizon model is hard to solve. The reason is that, since the information is long-lasting, the price conjecture (8) is not stationary anymore. The price coefficients are time-varying, depending on the present and future values of the meeting intensity λ. I briefly describe here the solution method for the finite version of the model. Details of the computations can be found in Appendix A.4.

I assume that the economy starts at time $t = -1$ and finishes at time $t = T + 1$. The trading dates are from $t = 1, .., T$. The meeting intensity λ is now time-varying. From t to $t + 1$ the meeting intensity is denoted by λ_{t+1}. Appendix A.1 describes the computations of the cross-sectional distribution μ_t of types when λ is time-varying.

The first dividend arising in this economy, D_2, is paid at time $t = 2$. The last dividend, D_{T+1}, is paid at time $t = T + 1$. The first dividend is equal to

$$D_2 = (1 - \kappa_d)\,\bar{D} + \kappa_d D_1 + \varepsilon_2^d,$$

where D_1 is a random number sampled from the unconditional distribution of the dividend process (1). The value of D_1 is learned by all the agents at time $t = 1$; no other private information exists about D_1 up to time $t = 1$.

Agents receive private information about the dividend innovation ε_2^d at time $t = -1$. This is as in the standard model, i.e. they receive information about the dividend innovation 3 periods ahead. During the 2 periods from $t = -1$ to $t = 1$ the agents meet with each other. The meeting intensities during these 2 periods are λ_0 and λ_1 respectively. At $t = 1$ the agents start trading based on their private information and to accommodate the supply shocks.

Supply shocks impact the economy at each trading date. The first supply shock, X_1, arises at time $t = 1$. The last supply shock is X_T. The first supply shock is equal to

$$X_1 = (1 - \kappa_x)\,\bar{X} + \kappa_x X_0 + \varepsilon_1^x,$$

where X_0 is an unobservable random number sampled from the unconditional distribution of the per capita supply process (2).

Having a time-varying λ complicates the solution method. If λ is assumed to be unobservable, uncertainty about it enters in the optimization problem (3). More precisely, the problem intervenes once one has to compute the individual expectation $\mathbb{E}_t^i[-e^{-\gamma \tilde{w}_{t+1}^i}]$. Since the one-step ahead λ can take two values with probabilities given by the Markov chain calibration from Section 3.7, the resulting first order condition does not provide a linear solution for x_t^i. The aggregation becomes then impossible and all the appealing features of the CARA-Gaussian framework disappear. Two assumptions help me to deal with this issue and maintain the setup tractable.

Assumption 1. *All the values of the meeting intensity λ up to time t are observed at time t.*

In light of the findings of Da et al. (2011) and Vlastakis and Markellos (2010), Assumption 1 seems reasonable. The investors' willingness to gather information can actually be proxied with frequency data from the main search engines.

Assumption 2. *When building his optimal trading strategy, each investor considers that all the future values of λ are equal to its unconditional mean.*

Assumption 2 helps me to deal with the problem of nonlinearity. If the agents consider that the future values of λ are given by the unconditional mean (computed from the Markov chain calibration of Section 3.7), then the optimization problem becomes linear again. I consider this a reasonable price to pay for the resulting analytical tractability.[7]

Another way to deal with this difficulty would be to assume that the agents have perfect foresight of λ. That is, λ is assumed to be time-varying but deterministic. Additionally, one could consider that all investors predict the same value of λ for future periods, using the Markov Chain parameters given above. The results obtained in separate calculations for these two cases are very similar. It turns out in all alternative numerical simulations that what matters most are the past values of λ, and not its future values.

To show the results, I consider a model with monthly data and an horizon of 60 months. It is assumed that the meeting intensity follows the same path as in Figure 7. Further details of the computations are reported in Appendix A.4.

Figure 8 shows the resulting volatility path for the first 2 years. The solid line depicts the dollar returns volatility, computed in (20), and can be seen as an impulse response after changes in the meeting intensity.[8]

As expected, the volatility increases with the search intensity. However, once the search intensity goes down, the volatility remains high for two more periods. The pattern of the volatility is dictated only partially by the moves in the information percolation speed. Volatility goes up synchronously with the search intensity but goes down at a slower rate.

The intuition behind this mechanism is straightforward. Once the search intensity goes up, the agents accumulate on average more signals about the future dividends. The *quantity* of their signals increases. This produces an increase in the volatility. Because

[7] In related research (Andrei and Hasler, 2010) we consider a general equilibrium model where the agents filter an unobservable fundamental by observing a public signal. The correlation between this public signal and the fundamental is stochastic and observable only up to the present. In that case, we still manage to get a closed form solution of the equilibrium price. However, the price to pay for this simplification is that in Andrei and Hasler (2010) prices do not have anymore an informational role.

[8] A similar path arises for the rates of return volatility. Given that the model has a finite horizon, it is more suitable in this case to show the dollar returns volatility. The rates or return volatility is influenced by the scaling by prices. The interpretations of its dynamics could be misleading in the finite horizon version of the model.

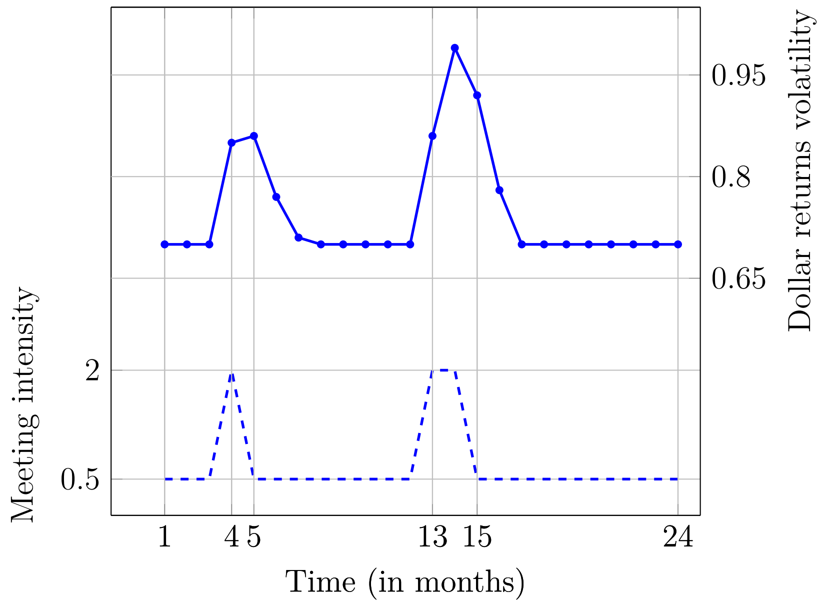

Figure 8: Volatility Clustering.
The dashed line shows a sample path of the meeting intensity λ_t, simulated with the Markov chain parameters from Section 3.7. The solid line shows the response of the rates of return volatility. The rates of returns are computed as in (20). The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

the information is long-lived, agents use it for several periods. Thus, even if the search intensity goes down, the agents exchange at further meetings a large amount of signals. This keeps the volatility high for a few more periods.

Two crucial elements produce this effect: the long-lived information and the word-of-mouth communication. First, the long-lived characteristic of the information is important because it allows agents to exchange information and trade more than once. Second, the word-of-mouth communication makes information more and more relevant as dividends approach payment dates. Without information percolation the persistence effect on volatility would disappear.

An additional result arising from the clustering of the volatility is the excess kurtosis of the unconditional distribution of the stock returns. While the conditional dollar returns as expressed in (20) are normally distributed, the excess kurtosis of the unconditional dollar returns computed for the simulated data of Figures 7 and 8 is systematically and significantly greater than zero (see, e.g., Bai, Russell, and Tiao, 2003).

4.2 Supporting Evidence

The last section assumes that the meeting intensity is time varying and proposes a Markov Chain process with parameters given in Table 2. However, the results go through

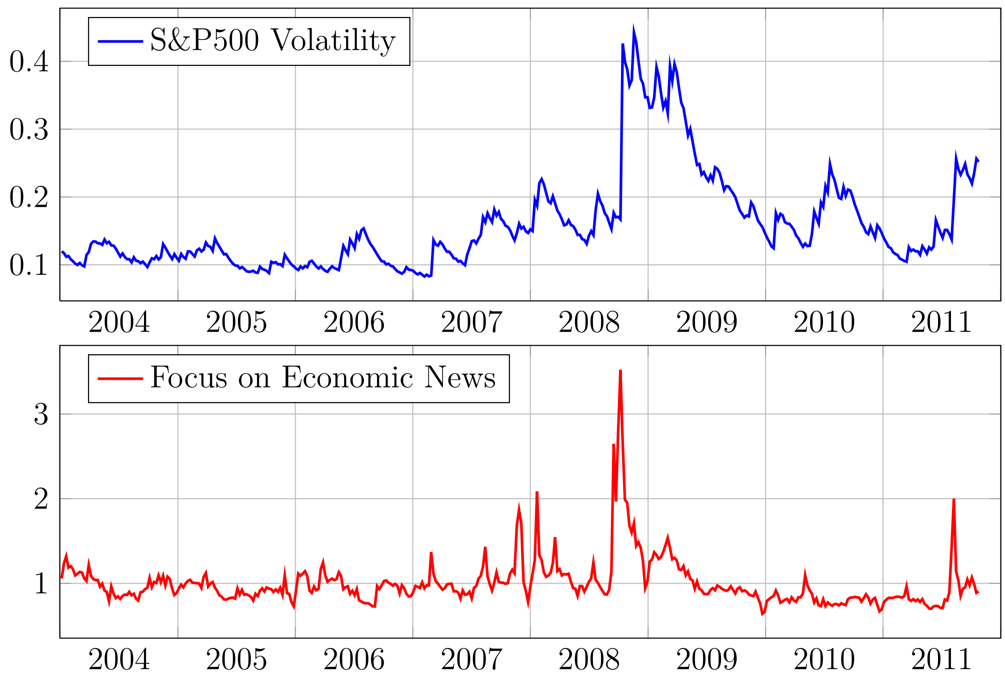

Figure 9: Focus on economic news and S&P500 Volatility.
The lower panel depicts a demeaned value weighted search index on financial and economic news from 2004 to 2011, at weekly frequency. The upper panel depicts the realized S&P500 annualized volatility from 2004 to 2011, resulted from a GARCH(1,1) estimation on weekly data.

with a wide range of parameters for the dynamics of λ.

For the calibration of the transition matrix in Table 2 I proceed as follows. Using insights from Da et al. (2011) and Vlastakis and Markellos (2010), I build from Google search data an index that I call "Focus on Economic News." It is depicted in the lower panel of Figure 9. This index is constructed using Google search volumes at weekly frequency on groups of words like "financial news," "economic news," "Wall Street Journal," "Financial Times," "CNN Money," "Bloomberg News," etc.[9] Other similar words in several combinations are used with always the same results. The resulted index reflects agents willingness to gather information. We observe that large peaks arrise at several key moments, like, for example, the Lehman Brothers bankruptcy in September 2008. Furthermore, I assume that agents' willingness to meet and agents willingness to gather information are related. Therefore, I perform a Markov Chain estimation based on this attention index. Table 2 is the result of this estimation.[10]

[9]More precisely, the index depicted in Figure 9 is built based on the following combination of words: "financial news," "economic news," "Wall Street Journal," "Financial Times," "CNN Money," "Bloomberg News," "S&P500," "us economy," "stock prices," "stock market," "NYSE," "NASDAQ," "DAX," and "FTSE." Other similar words in several combinations are used with almost identical results.

[10]For the Markov Chain estimation I used the package *MS Regress*, created by Perlin (2010).

The upper panel of Figure 9 depicts the volatility of the S&P500. It is, as expected, time-varying. It rises very fast at times, decaying only slowly after. Inspection of both panels reveals simultaneous upper jumps in both the focus on economic news and the volatility. But, the focus on economic news goes down faster than the volatility. A raw two state Markov chain estimation on monthly data for stock returns reveals that the probability of staying in a high volatility state is 60% versus 20% for the attention index. Clearly, the volatility goes down slower than investors' attention.

The apparent synchronous upper jumps but asynchronous descents observed in Figure 9 can be explained by the information percolation. As showed in Section 4.1, only a transitory shock of the meeting intensity can induce a long-lasting effect on the volatility. Empirically, it has been shown (see, for a recent reference, Berger, Chaboud, and Hjalmarsson, 2009) that the persistence in the information flow is not large enough to capture the persistence in the volatility. In the present model, although the generation of information is i.i.d., the information percolation generates volatility dynamics consistent with empirical findings.

5 Trading Volume and Volatility

A sizeable literature documents the link between information flow and measures of market activity, such as trading volume and return volatility (see, for example, French and Roll, 1986; Ross, 1989; Andersen, 1996; Andersen and Bollerslev, 1997). The widespread hypothesis is that the rate of arrival of information in the market drives both return volatility and trading volume. Clark (1973) is the first to introduce the "Mixture of Distribution Hypothesis" (MDH), i.e. the joint dependence of both volume and returns on a latent information process. Lamoureux and Lastrapes (1990) observe that the inclusion of trading volume in the variance process makes the GARCH coefficients not significant, suggesting that volume and volatility are driven by a common factor, and thus confirming the MDH. Andersen (1996) further develops The MDH into the "Modified MDH" from a stylized market microstructure framework. In his model, the information arrival is approximated by a Poisson process and governs the dynamic features of returns and volume. The imposition of a conditional Poisson rather than normal distribution results in a model which fits reasonably the data, although the more traditional version of the MDH—assuming trading volume to be normally distributed—is firmly rejected.

The information percolation and the Modified MDH present a similarity. In both cases, the information flow follows a Poisson process. This motivates my questioning whether the information percolation could drive both volume and volatility.

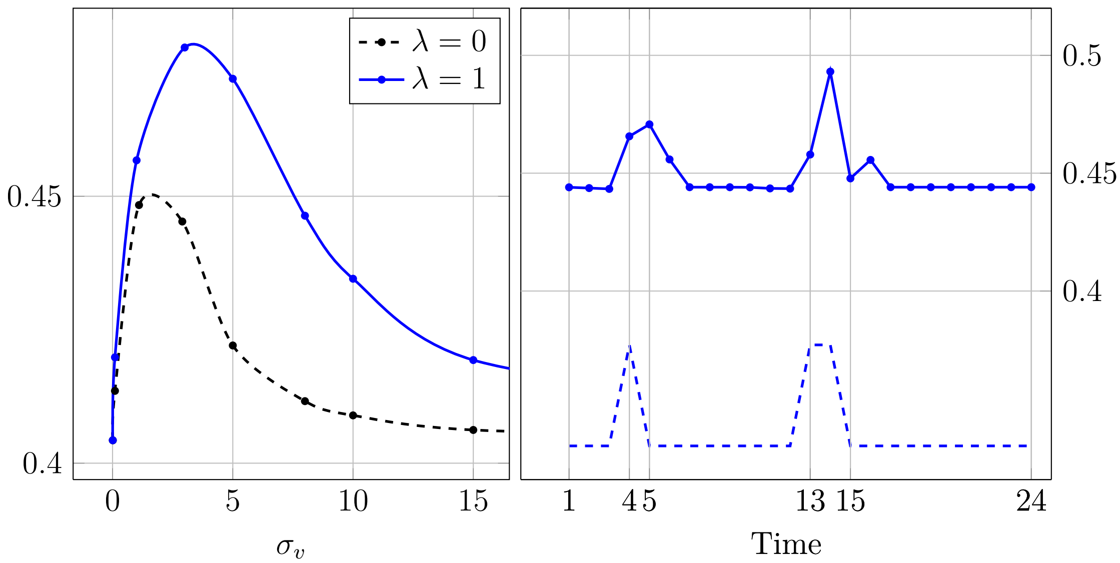

Figure 10: Trading Volume and Information Percolation.
The left panel plots the trading volume in an infinite horizon economy for different values of the signal noise σ_v. There are two lines: the black dashed line corresponds to the case without information percolation, while the blue solid line is for the case $\lambda = 1$. The right panel shows the dynamics of the trading volume for a given path of the meeting intensity λ. The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

I follow He and Wang (1995) and define the trading volume as the cross-sectional average of the absolute change in investors' positions over time:

$$V_t = \frac{1}{2}\int_i \left| x_t^i - x_{t-1}^i \right| di + \frac{1}{2}\left| X_t - X_{t-1} \right|$$

The computations are reported for convenience in Appendix A.6. The left panel of Figure 10 depicts the average volume for different values of the signal noise σ_v. There are two lines: the black dashed line corresponds to the case without information percolation, while the blue solid line is for the case $\lambda = 1$. The trading volume is larger for intermediate values of the signal noise, because of the increased disagreement. Hence, investors trade more aggressively for higher values of λ because their private information is more precise. In other words, the trading volume is higher in an economy with a higher meeting intensity.

I compute the dynamics of the trading volume for the same pattern of the meeting intensity as in Section 4.1. For this, I consider the case of a similar economy in which all agents having the same number of signals, $\{\bar{n}_{t,1}, \bar{n}_{t,2}\}$. This is because the discussion related to heterogeneity of investors is not necessary in this case. Details of the computations are reported for convenience in Appendix A.6. The results are shown in the right panel of Figure 10. The trading volume increases at times of higher meeting intensity, then it remains high for more periods than λ, as it is the case for the stock market volatility. This persistent effects in the dynamics of trading volume arise for two reasons. First, since investors posses on average a large amount of signals once the word-of-mouth communication intensifies, they trade more aggressively, and they will do so as long as the information remains relevant for future dividends. Second, since investors use price movements as information on which to make trading decisions, the large price movements produced by information percolation will cause large trading volume. Hence the joint dependence of the trading volume and the volatility on the same underlying process, i.e., the information flow to the market.

6 Additional Implications

6.1 The Term Structure of Risk

Which dividends drive the volatility? By recovering prices of zero-coupon equity (dividend strips) on the aggregate stock market, Binsbergen et al. (2010) have discovered that a large amount of the volatility is concentrated in the short-term, challenging leading asset pricing models that take the *equilibrium approach*.[11] Only one study (Lettau and Wachter, 2007), using the *stochastic discount factor approach*, predicts exactly the feature highlighted by Binsbergen et al. (2010).[12] Since Lettau and Wachter (2007) exogenously specify the joint dynamics of cash flows and of the stochastic discount factor, it is not a full-fledged equilibrium model. An important next step is to build the microfoundations that can give rise to their specification.

In this section I show that the information percolation can provide the feature highlighted by Binsbergen et al. (2010)—i.e., the volatility is driven by short-maturity dividends. First, as dividends approach payment dates, agents accumulate an increasing amount of information, making the short-term asset increasingly sensitive to supply shocks and future dividend shocks. Second, as agents accumulate more information, they might disagree more on the future values of dividends; in other words, information percolation implies a term structure of disagreement. These two mechanisms drive the shape of the term structure of volatility, consistent with the recent empirical findings of Binsbergen et al. (2010).

Notice that the price can be expressed in terms of a stochastic discount factor. The

[11] Under the equilibrium approach, asset returns are endogenously determined by the form of preferences and the process for aggregate consumption. Representative models are Campbell and Cochrane (1999), Bansal and Yaron (2004), or Gabaix (2008).

[12] This approach takes the reverse engineering path, by specifying directly a stochastic discount factor for the economy, allowing for better flexibility in matching asset prices. Representative models are Lettau and Wachter (2007) and Lettau and Wachter (2009).

optimization problem (3) leads to the asset pricing equation

$$P_t = \mathbb{E}_t^i \left[\frac{e^{-\gamma \tilde{w}_{t+1}^i}}{R\mathbb{E}_t^i[e^{-\gamma \tilde{w}_{t+1}^i}]} \left(P_{t+1} + D_{t+1}\right) \right].$$

This expectation can still be computed in closed form, because both $\tilde{w}_{t+1}^i$ and $(P_{t+1} + D_{t+1})$ are Gaussian. The pricing kernel corresponding to investor i is

$$M_{t+1}^i = \frac{e^{-\gamma \tilde{w}_{t+1}^i}}{R\mathbb{E}_t^i[e^{-\gamma \tilde{w}_{t+1}^i}]} = \frac{e^{-\gamma x_t^i (P_{t+1}+D_{t+1})}}{R\mathbb{E}_t^i[e^{-\gamma x_t^i (P_{t+1}+D_{t+1})}]}.$$

With dispersed information, each agent will have a different pricing kernel. Despite heterogeneity, the computation of all the individual expectations and the proper replacement of the individual optimal strategies x_t^i lead to the same value, P_t.

As in Lettau and Wachter (2007), the building blocks of the long-lived asset in this economy are "zero-coupon" equity, or dividend strips. I denote by $P_{t,n}$ the price of an asset that pays the aggregate dividend n periods from now. By definition, the price of the entire asset, P_t, is equal to $\sum_{n=1}^{\infty} P_{t,n}$. The aim is to compute the volatility of each term of the sum. For this, I compute the average valuation for each dividend strip across the population of agents as follows

$$P_{t,n} = \hat{\mathbb{E}}_t \left[M_{t+1} \hat{\mathbb{E}}_{t+1} \left[M_{t+2} ... \hat{\mathbb{E}}_{t+n-2} \left[M_{t+n-1} \hat{\mathbb{E}}_{t+n-1} \left[M_{t+n} D_{t+n} \right] \right] \right] \right],$$

where M_{t+j}, $j = 1...n$, denotes the pricing kernel of a representative agent, whose conditional expectation and conditional variance of $(P_{t+j+1} + D_{t+j+1})$ are $\mu \equiv \widehat{\mathbb{E}}_{t+j} \left(P_{t+j+1} + D_{t+j+1}\right)$ and $\sigma^2 \equiv 1/\bar{K}_{t+j}$ respectively (a representative agent with average beliefs). Notice the recursive nature of expectations (higher order expectations, Bacchetta and Wincoop, 2008), arising because the law of iterated expectations does not generally hold for market expectations when investors have different information sets. The Euler equation stated in a recursive form writes

$$P_{t,n} = \widehat{\mathbb{E}}_t \left[M_{t+1} P_{t+1,n-1} \right], \tag{21}$$

with boundary condition $P_{t,0} = D_t$.

To compute $P_{t,n}$, one needs to start recursively at $t + n - 1$ and use (21). I compute first $P_{t+n-1,1}$, then I compute $P_{t+n-2,2}$, and so on. The equation (21) can be computed directly using the Gaussian assumption. The details are in the Appendix Section A.5. Once the expectation is computed, the volatility of dividend strips, $\sigma(P_{t+1,n-1} - RP_{t,n})$, can be computed in closed form. I turn now to the analysis of this volatility.

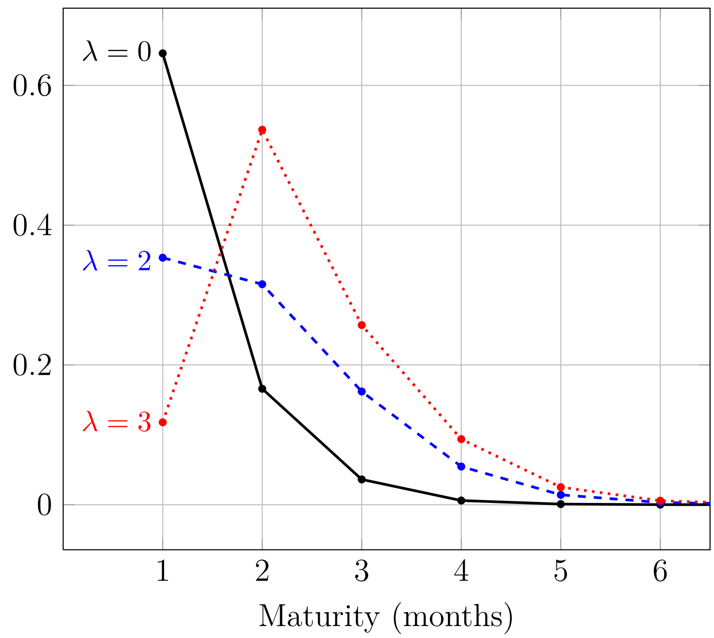

Figure 11: Return Volatility of Dividend Strips.
Volatility of dividend strip returns, $\sigma(P_{t+1,n-1} - RP_{t,n})$, for $n = 1$ to 6. The black solid line represents the case with no information percolation. The blue dashed line represents the case with $\lambda = 2$, and the red dotted line represents the case with $\lambda = 3$. The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

Risk in the Short Term

The results are presented in Figure 11. There are 3 lines showing respectively the cases "no information percolation" $\lambda = 0$ (black solid line), and "information percolation" at two levels, $\lambda = 2$ (blue dashed line) and $\lambda = 3$ (red dotted line). The lines can be interpreted as term structures of the volatility. I chose a high value of the meeting intensity for the red dotted line to illustrate the hump-shaped pattern dictated by the similar pattern in disagreement. That is, for high levels of the meeting intensity, the disagreement regarding the next period dividend goes down, because agents have now on average very reliable information about that dividend. However, volatilities of returns on the second and subsequent dividends increase, being driven by a higher disagreement.

The short-run dividend strips feature a relatively higher level of return volatility, which decays fast with maturity. The information percolation clearly amplifies the short-run volatility.

As dividends approach payment dates, the information becomes more and more relevant, making the short-term asset increasingly sensitive to both future dividend shocks and supply shocks. The effect is magnified naturally by the information percolation. The information percolation increases the precision of the signals about future dividends

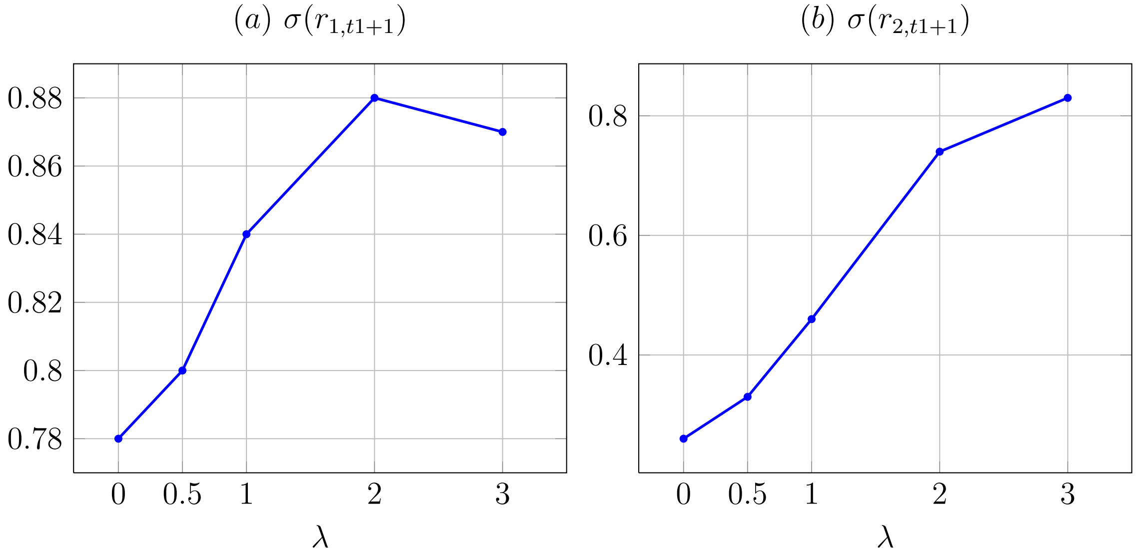

Figure 12: Information Percolation and Short-Term Volatility.
Volatility of the short term assets returns, computed as in (22) (left panel) and (23) (right panel). The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

proportionally to their timing: closer dividends will get a higher increase than far-away dividends. Most of the volatility is then concentrated in the short run, consistent with Binsbergen et al. (2010).

Furthermore, I denote the short-term asset by $\mathcal{P}_{t,n_1,n_2}$, with $1 \leq n_1 \leq n_2$. This asset entitles the holder to receive the dividends arising from $t + n_1$ to $t + n_2$. I consider two dividend strategies inspired from Binsbergen et al. (2010). The first strategy consists in buying the short-term asset which pays the next 24 months of dividends. The dollar returns of this strategy are computed as follows:

$$r_{1,t+1} = \sigma \left(D_{t+1} + \mathcal{P}_{t+1,1,23} - R\mathcal{P}_{t,1,24} \right) \tag{22}$$

The second strategy is called dividend steepener, for which $n_1 > 1$. This strategy does not involve dividend payments until time $t + n_1$. For this example, I choose $n_1 = 2$ and $n_2 = 24$. The dollar returns of this strategy are computed as follows:

$$r_{2,t+1} = \sigma \left(\mathcal{P}_{t+1,1,23} - R\mathcal{P}_{t,2,24} \right) \tag{23}$$

The volatilities of returns of the two strategies are depicted in Figure 12. In an economy with higher meeting intensity, the short-term asset is more sensitive to future dividend shocks. Thus, the information percolation increases the volatility of the short-term asset, for both strategies.

6.2 Heterogeneous Trading Strategies

Recent empirical literature documents various patterns of trade that can be interpreted as evidence of word-of-mouth communication. For example, Hong et al. (2005) find that covariance of trades among fund managers is higher if they are situated in the same city. Communications via shared education networks help fund managers make excess returns by over-investing in firms run by their former classmates (Cohen et al., 2008). In Massa and Simonov (2011), formation of close personal relationships between college alumni influence portfolio choice—investors invest in the same stocks in which their former classmates do. Other papers provide strong evidence that language, social networks, and geographical proximity influence portfolio choices.[13]

Because it relies on heterogeneity of investors' information, the rational expectations model is particularly suitable to explain the above findings. For instance, Brennan and Cao (1997) build an international finance setup in which domestic investors have an informational advantage relative to foreign investors. This makes domestic investors act as contrarians, while foreign investors act as trend followers. Their results are confirmed by the data. In other theoretical models, Watanabe (2008) and Colla and Mele (2010) obtain the same result—less informed agents are trend-followers, while better informed agents are contrarians.

In the context of the present model, the information percolation generates a pronounced heterogeneity in the investors' information endowments. Agents receive always one signal for the 3-periods ahead dividend. As they meet with each other, they progressively become heterogeneous with respect to their number of signals. This creates a natural setup to examine the different patterns of trade that emerge for each type of agents. The results show that better informed investors tend to act as contrarians, while less informed investors tend to act as trend-followers. While Watanabe (2008) exogenously divides the total mass of agents in J groups in order to show the same result, here the result arises naturally from the information percolation mechanism.

To show this, I proceed as in Watanabe (2008) and consider the trading strategy of each group of investors. In my case, the different investor groups simply arise from the information percolation. They are characterized by the couple $\{n^i_{t,1}, n^i_{t,2}\}$ and their proportion is $\mu\left(n^i_{t,1}, n^i_{t,2}\right)$, following Section 3.3.

In order to analyze investors' trading strategies associated with differential information, it is necessary to consider investors' positions net of per capita supply shock. That is, one has to disentangle trading based on differential information and trading to

[13]Feng and Seasholes (2004) find that in the Chinese stock market the geographical distance has an effect on trading—geographically close investors have positively correlated trades, while distant investors have negatively correlated trades. Other related papers are Grinblatt and Keloharju (2001), Hong et al. (2004), Ivkovic and Weisbenner (2005), and Shive (2010).

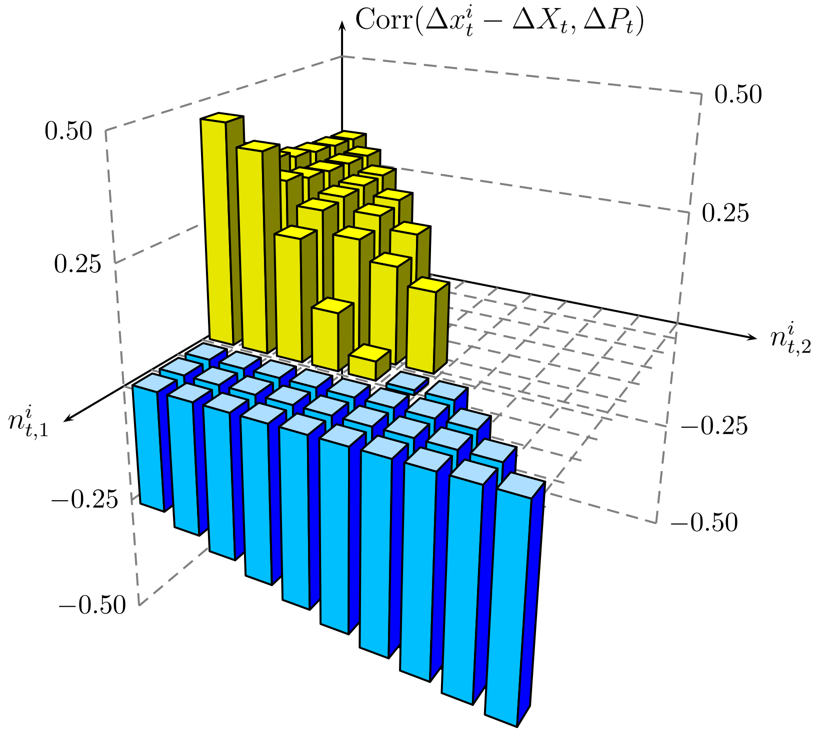

Figure 13: Heterogeneous Trading Strategies.

This graph is the counterpart of of Figure 2, showing the trading strategies by investor types (number of signals gathered). The population of investors is dividend in two camps: contrarians (blue bars) and trend-followers (yellow bars). The parameter values are: $R = 1.004$, $\gamma = 1$, $\sigma_d = 0.628$, $\sigma_x = 0.358$, $\sigma_v = 5$, $\bar{D} = 0.224$, $\bar{X} = 0.176$, $\kappa_d = 0.129$, $\kappa_x = 0$.

accomodate the supply shock (market-making), as done in He and Wang (1995) and Brennan and Cao (1997). For this purpose, I further assume that the contribution of noise traders of type $\{n_{t,1}^i, n_{t,2}^i\}$ to the aggregate supply shock X_t is $\mu\left(n_{t,1}^i, n_{t,2}^i\right) X_t$. After adding the noise trades to those of rational investors, I compute the aggregate trading volume for each group of investors. The details of the computations are in Appendix A.6.

Figure 13 depicts the correlation between the informational trading strategy of each group of investors, $\Delta x_t^i - \Delta X_t$, and the contemporaneous price difference, $\Delta P_t \equiv P_t - P_{t-1}$. The investors having a few number of signals tend to be trend-followers, while the investors who gathered a relatively more important number of signals are contrarians.

This stylized fact, documented by numerous empirical studies, might be of particular importance when trying to explain the home bias (documented first by French and Poterba, 1991) and the distance influence on investment decisions (documented first by Coval and Moskowitz, 1999).[14] Particularly, it is a natural assumption

[14]The home bias puzzle is a long standing empirical problem of the CAPM. It describes the fact that

that the frequency of meetings among people is higher within their country than outside. In the context of the present model, two levels of the meeting intensity—one for meeting home investors (larger) and other for meeting foreign investors (smaller)—might then help accumulate an informational advantage at home. This may generate different portfolio holdings and patterns of trade consistent with the aforementioned empirical literature.

7 Concluding Remarks

Word-of-mouth communication is an innate feature of humans, helping them to process and aggregate knowledge. This paper is an attempt to show that word-of-mouth communication impacts both the level and the dynamics of stock price volatility. It does so by interacting with the price formation mechanism—an additional yet different tool of aggregating knowledge. Word-of-mouth communication and the price formation mechanism define two sides of the same coin: the process through which information is elaborated, transmitted, and aggregated.

The first implications is that word-of-mouth communication amplifies the volatility of asset returns. Second, episodes of intense word-of-mouth communication, although transitory, can generate persistent volatility. This arises because information is long-lived—once agents gather a large amount of information, they spread it through word-of-mouth and trade based on it during subsequent trading sessions. Third, dividends from the recent future are prone to interpersonal discussions, compared with dividends situated far into the future. Therefore, the volatility increases mostly in the short-term and relatively less in the long-term. Forth, word-of-mouth communication pushes investors to trade more aggressively, increasing the trading volume. Therefore, word-of-mouth communication drives both trading volume and volatility. Fifth, the random accumulation of information generates heterogeneity in investors' information endowments, resulting in patterns of trade consistent with empirical findings.

This paper raises several potentially interesting questions for future research. First, word-of-mouth communication may generate fads or rumors instead of real information. The present setup offers a tractable framework for measuring the consequence of rumors for prices. Second, in the context of a model with fads and rumors, what is the effect of additional public signals, such as earnings announcements? Probably the transparency and accuracy of public information is beneficial for financial markets, minimizing the

investors in most countries hold only a small share of foreign equity, although they could greatly benefit from international diversification. The home bias puzzle is still sizable today (see Sercu and Vanpee, 2007, for one of the latest reviews) despite the fact that information about stock markets is diffused globally and that trading in international stocks is increasingly easy.

effects of low-quality rumors. Third, what if agents can optimally choose the meeting intensity? This would allow to relate the meeting intensity with the business cycle conditions: it might be optimal for agents to look for more information during bad economic times, as we observe empirically. Fourth, how does the information percolation interacts with higher order beliefs? As the percolation generates overreliance to public information, the effect of higher order beliefs might be magnified, disconnecting the price further away from its fundamental value. Finally, in an international finance context, the effect of the information percolation can be relevant for exchange rate volatility and for other empirical anomalies such as the home equity bias. I leave further work along these lines for future research.

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A Appendix

A.1 Information Percolation

I describe here the solution method in the general case, when λ is time-varying. Following Duffie et al. (2009), the cross-sectional distribution satisfies the following Boltzmann equation

$$\frac{d}{dt}\mu_t = \lambda_t \mu_t * \mu_t - \lambda_t \mu_t. \tag{A.1}$$

The simplest way to solve (A.1) is to start at $t-2$, when each agent receives one signal about D_{t+1}. Between $t-2$ and $t-1$ agents meet with intensity λ_{t-1}. At any time $\tau \in [0,1]$, call the distribution of the number of signals $\varphi_\tau(m)$. This is an univariate probability distribution. The corresponding Boltzmann equation is

$$\frac{d}{d\tau}\varphi_\tau(m) = \lambda_{t-1} \sum_{i=1}^{m-1} \varphi_\tau(i)\varphi_\tau(m-i) - \lambda_{t-1}\varphi_\tau(m),$$

which can be solved recursively, starting with $m = 1$:

$$\frac{d}{d\tau}\varphi_\tau(1) = -\lambda_{t-1}\varphi_\tau(1),$$

with boundary condition $\varphi_0(1) = 1$. The solution is $\varphi_\tau(1) = e^{-\tau\lambda_{t-1}}$. Having now $\varphi_\tau(1)$, it can be replaced in the equation of $\varphi_\tau(2)$:

$$\frac{d}{d\tau}\varphi_\tau(2) = \lambda_{t-1}\varphi_\tau(1)^2 - \lambda_{t-1}\varphi_\tau(2),$$

with boundary condition $\varphi_0(2) = 0$. This gives $\varphi_\tau(2) = e^{-2\tau\lambda_{t-1}}(e^{\tau\lambda_{t-1}} - 1)$. By going further it can be easily seen that the general formula is

$$\varphi_\tau(m) = e^{-m\tau\lambda_{t-1}}(e^{\tau\lambda_{t-1}} - 1)^{m-1}. \tag{A.2}$$

It can be verified that

$$\sum_{m=1}^{\infty} \varphi_\tau(m) = 1$$
$$\sum_{m=1}^{\infty} m\varphi_\tau(m) = e^{\tau\lambda_{t-1}}.$$

The distribution of signals at $\tau = 1$ will be used as a boundary condition in what follows.

Go now at time $t-1$. Each agent i receives one signal about D_{t+2} and has m^i signals about D_{t+1}, with the probability density function of m^i given by (A.2). They share these signals between $t-1$ and t with intensity λ_t. At any time $\tau \in [0,1]$, call the distribution of the number of signals $\mu_\tau(n_1^i, n_2^i)$, with n_1^i the number of signals about D_{t+1} and n_2^i the number of signals about D_{t+2}. This is a bivariate probability distribution. The corresponding Boltzmann equation is (A.1). It can be solved recursively in the same way. Start with

$$\frac{d}{d\tau}\mu_\tau(1,1) = -\lambda_t\mu_\tau(1,1),$$

with boundary condition $\mu_0(1,1) = \varphi_1(1)$, and $\varphi_1(1)$ has been obtained in the previous step.

Once $\mu_\tau(1,1)$ is obtained, it can be replaced further away in the iterations as shown before. The general equation (for $n_2^i > 1$) is

$$\frac{d}{d\tau}\mu_\tau(n_1^i, n_2^i) = \lambda_t \sum_{i=1}^{n_1^i-1} \sum_{j=1}^{\min\{i, n_2^i-1\}} \mathbb{1}_{\{n_2^i - j \leq n_1^i - i\}} \mu_\tau(i,j)\mu_\tau(n_2^i - j, n_1^i - i) - \lambda_t \mu_\tau(n_1^i, n_2^i),$$

with boundary condition $\mu_0(n_1^i, n_2^i) = 0$. Solving this equation recursively gives (7).

A.2 Learning

The matrices $\mathbb{A}$ and $\mathbb{B}$ are

$$\mathbb{A} = \begin{pmatrix} a_3 & a_2 & a_1 \\ a_2 & a_1 & 0 \\ a_1 & 0 & 0 \end{pmatrix}, \; \mathbb{B} = \begin{pmatrix} b_3 & b_2 & b_1 \\ b_2 & b_1 & 0 \\ b_1 & 0 & 0 \end{pmatrix}.$$

Write the errors of the signals in (10) as

$$\begin{pmatrix} \mathbb{B}\epsilon_t^x \\ \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix} = \begin{pmatrix} b_3 & b_2 & b_1 & 0 & 0 & 0 \\ b_2 & b_1 & 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{t-2}^x \\ \varepsilon_{t-1}^x \\ \varepsilon_t^x \\ \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{vi} \\ \varepsilon_{t,1}^{vi} \end{pmatrix}, \tag{A.3}$$

and call the matrix on the right hand side Θ. Since the variance of $\varepsilon_{t,1}^{wi}$ and of $\varepsilon_{t,2}^{wi}$ depend on the number of signals that each investor gathered, the variance of the errors of the signals in (10), $\mathbb{R}^i$, is heterogeneous across investors—it depends on the couple $\{n_{t,1}^i, n_{t,2}^i\}$. More precisely, the covariance of the vector from the right hand side of (A.3) is equal to

$$\Sigma^i = \begin{pmatrix} \sigma_x^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_x^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_x^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sigma_v^2}{n_{t,2}^i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\sigma_v^2}{n_{t,2}^i} \end{pmatrix}$$

and thus $\mathbb{R}^i = \Theta\Sigma^i\Theta^\top$.

To prepare the setting for the projection theorem everything can be grouped under the following form:

$$\begin{pmatrix} \varepsilon_{t+1}^d \\ \varepsilon_{t+2}^d \\ \varepsilon_{t+3}^d \\ \mathbb{P}t \\ v_t^i \\ W_t^i \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} \mathbf{0}_{9\times 1} \end{pmatrix}, \left(\begin{array}{c|c} \sigma_d^2 \mathbb{I}_3 & \sigma_d^2 \mathbb{H}^\top \\ \hline \sigma_d^2 \mathbb{H} & \sigma_d^2 \mathbb{H}\mathbb{H}^\top + \mathbb{R}^i \end{array}\right)\right], \tag{A.4}$$

where $\mathbf{0}_{9\times 1}$ is a 9×1 vector of zeros and $\mathbb{I}_3$ is the identity matrix of dimension 3.

The projection theorem states that if

$$\left(\begin{array}{c}\theta\\ s\end{array}\right)\sim\mathcal{N}\left[\left(\begin{array}{c}\mu_\theta\\ \mu_s\end{array}\right),\left(\begin{array}{cc}\Sigma_{\theta\theta} & \Sigma_{\theta s}\\ \Sigma_{s\theta} & \Sigma_{ss}\end{array}\right)\right],$$

then

$$\begin{aligned}\mathbb{E}[\theta/s] &= \mu_\theta+\Sigma_{\theta s}\Sigma_{ss}^{-1}(s-\mu_s)\\ \mathrm{Var}[\theta/s] &= \Sigma_{\theta\theta}-\Sigma_{\theta s}\Sigma_{ss}^{-1}\Sigma_{s\theta}.\end{aligned}$$

Direct application of the projection theorem to (A.4) leads to

$$\begin{aligned}\mathbb{E}_t^i\epsilon_t^d &= \mathbb{M}^i\left(\begin{array}{c}\mathbb{P}_t\\ v_t^i\\ W_t^i\end{array}\right)\\ \mathrm{Var}_t^i\epsilon_t^d &= \sigma_d^2\left(\mathbb{I}_3-\mathbb{M}^i\mathbb{H}\right),\end{aligned}$$

with $\mathbb{M}^i=\sigma_d^2\mathbb{H}^\top\left(\sigma_d^2\mathbb{H}\mathbb{H}^\top+\mathbb{R}^i\right)^{-1}$.

The last equalities in (11) and (12) can be obtained by using of the Woodbury matrix identity. For the conditional variance:

$$\begin{aligned}\sigma_d^2\left(\mathbb{I}_3-\mathbb{M}^i\mathbb{H}\right) &= \sigma_d^2\mathbb{I}_3-\sigma_d^2\mathbb{I}_3\mathbb{H}^\top\left(\mathbb{R}^i+\mathbb{H}\sigma_d^2\mathbb{I}_3\mathbb{H}^\top\right)\mathbb{H}\sigma_d^2\mathbb{I}_3\\ &= \left(\frac{1}{\sigma_d^2}\mathbb{I}_3+\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\right)^{-1},\end{aligned}\tag{A.5}$$

and for the conditional expectation:

$$\begin{aligned}\mathbb{M}^i &= \sigma_d^2\mathbb{H}^\top\left[(\mathbb{R}^i)^{-1}-(\mathbb{R}^i)^{-1}\mathbb{H}\left(\frac{1}{\sigma_d^2}\mathbb{I}_3+\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\right)^{-1}\mathbb{H}^\top(\mathbb{R}^i)^{-1}\right]\\ &= \left[\sigma_d^2-\sigma_d^2\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\left(\frac{1}{\sigma_d^2}\mathbb{I}_3+\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\right)^{-1}\right]\mathbb{H}^\top(\mathbb{R}^i)^{-1}\\ &= \left(\mathrm{Var}_t^i\epsilon_t^d\right)\mathbb{H}^\top(\mathbb{R}^i)^{-1}.\end{aligned}$$

The last equality is obtained by using (A.5) and recognizing that

$$\left[\sigma_d^2-\sigma_d^2\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\left(\frac{1}{\sigma_d^2}\mathbb{I}_3+\mathbb{H}^\top(\mathbb{R}^i)^{-1}\mathbb{H}\right)^{-1}\right]\left(\mathrm{Var}_t^i\epsilon_t^d\right)^{-1}=\mathbb{I}_3.$$

A.3 Equilibrium

I restate the equation (13) here for convenience

$$\frac{1}{\gamma}\left(\int_0^1\frac{\mathbb{E}_t^i\left(P_{t+1}+D_{t+1}\right)}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)}di-RP_t\int_0^1\frac{1}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)}di\right)=X_t\tag{A.6}$$

where X_t can be written as

$$X_t = \left(1-\kappa_x^3\right)\bar{X} + \kappa_x^3 X_{t-3} + \begin{pmatrix} \kappa_x^2 & \kappa & 1 \end{pmatrix} \epsilon_t^x.$$

The second integral represents the average precision of the entire population of investors at time t, that shall be denoted hereafter by $\bar{K}_t$:

$$\begin{aligned}\bar{K}_t &= \int_0^1 \frac{1}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)} di \\ &= \sum_{n_{t,1}^i=1}^{\infty} \sum_{n_{t,2}^i=1}^{\infty} \mu\left(n_{t,1}^i, n_{t,2}^i\right)\left(a_1^2\sigma_d^2 + b_1^2\sigma_x^2 + \psi\left(\mathrm{Var}_t^i \epsilon_t^d\right)\psi^\top\right)^{-1}.\end{aligned} \tag{A.7}$$

By using this result and equation (16) we can turn to the first integral:

$$\begin{aligned}\int_0^1 \frac{\mathbb{E}_t^i\left(P_{t+1}+D_{t+1}\right)}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)} di =& \bar{K}_t\left(f(\bar{D}, D_t, \bar{X}, X_{t-3}) + b^*\mathbb{B}^{-1}\mathbb{A}\epsilon_t^d + b^*\epsilon_t^x\right) \\ &+ \int_0^1 \frac{\psi\mathbb{M}^i}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)} \begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix} di.\end{aligned}$$

By aggregation, one obtains

$$\int_0^1 \frac{\psi\mathbb{M}^i}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)} \begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix} di = \bar{L}_t \begin{pmatrix} \mathbb{P}_t \\ \bar{v}_t \\ \bar{W}_t \end{pmatrix}$$

The vector $\bar{L}_t$, of dimension (1×6), is equal to

$$\bar{L}_t = \sum_{n_{t,1}^i=1}^{\infty} \sum_{n_{t,2}^i=1}^{\infty} \mu\left(n_{t,1}^i, n_{t,2}^i\right)\psi\mathbb{M}^i\left(a_1^2\sigma_d^2 + b_1^2\sigma_x^2 + \psi\left(\mathrm{Var}_t^i \epsilon_t^d\right)\psi^\top\right)^{-1}.$$

Thus

$$\bar{L}_t \begin{pmatrix} \mathbb{P}_t \\ \bar{v}_t \\ \bar{W}_t \end{pmatrix} = \bar{L}_t\left(\mathbb{H}\epsilon_t^d + \mathbb{B}^*\epsilon_t^x\right),$$

where $\mathbb{B}^* \equiv \begin{pmatrix} \mathbb{B} \\ \mathbf{0}_{3\times 3} \end{pmatrix}$.

To summarize:

$$\begin{aligned}\int_0^1 \frac{\mathbb{E}_t^i\left(P_{t+1}+D_{t+1}\right)}{\mathrm{Var}_t^i\left(P_{t+1}+D_{t+1}\right)} di =& \bar{K}_t f(\bar{D}, D_t, \bar{X}, X_{t-3}) + \left(\bar{K}_t b^*\mathbb{B}^{-1}\mathbb{A} + \bar{L}_t\mathbb{H}\right)\epsilon_t^d \\ &+ \left(\bar{K}_t b^* + \bar{L}_t\mathbb{B}^*\right)\epsilon_t^x\end{aligned} \tag{A.8}$$

It remains now to replace (A.7) and (A.8) in the market clearing condition (A.6). The coefficients $\bar{\alpha}$, α, $\bar{\beta}$, β, a_j, and b_j, for $j = 1, 2, 3$, must solve the following equations:

1. Coefficient of $\bar{D}$:

$$[\bar{\alpha} + (\alpha+1)(1-\kappa_d)] - R\bar{\alpha} = 0$$

2. Coefficient of D_t:

$$(\alpha+1)\kappa_d - R\alpha = 0$$

3. Coefficient of $\bar{X}$:

$$\bar{K}_t[\bar{\beta} + \beta(1-\kappa_x)] - \bar{K}_t R\bar{\beta} - \gamma(1-\kappa_x^3) = 0$$

4. Coefficient of X_{t-3}:

$$\bar{K}_t\beta\kappa_x - \bar{K}_t R\beta - \gamma\kappa_x^3 = 0$$

5. Coefficient of ϵ_t^d:

$$\bar{K}_t b^* \mathbb{B}^{-1}\mathbb{A} + \bar{L}_t\mathbb{H} - \bar{K}_t R a = \mathbf{0}_{1\times 3}$$

6. Coefficient of ϵ_t^x:

$$\bar{K}_t b^* + \bar{L}_t\mathbb{B}^* - \bar{K}_t R b - \gamma \begin{pmatrix} \kappa_x^2 & \kappa_x & 1 \end{pmatrix} = \mathbf{0}_{1\times 3}$$

This system of 9 equations (items 5 and 6 are vector equations) is solved numerically. The algorithm is very efficient, because there is a very natural starting point. For this, I consider an economy in which all the agents have the same number of signals $\{\bar{n}_{t,1}, \bar{n}_{t,2}\}$, where $\bar{n}_{t,1}$ and $\bar{n}_{t,2}$ are the average numbers of signals computed in subsection 3.3. Giving this result as a starting point to the numerical algorithm makes the computation very efficient.

A.4 Finite Model

The solution method works as follows:

1. Consider the economy at time $t = 1$, and $\lambda_0 = \lambda_1 = 0.5$. Solve the model in this case (see below the conjecture for the price coefficients).
2. Move one period further at time $t = 2$. According to Figure 7, $\lambda_2 = 0.5$. Solve the model now by forcing the coefficients of the price P_1 to be fixed at the solution found at step 1.
3. Move one period further at time $t = 3$. According to Figure 7, $\lambda_3 = 0.5$. Solve the model now by forcing the coefficients of the prices P_1 and P_2 to be fixed at the solutions found at steps 1 and 2.
4. Go on with these iterations up to time $t = T$. Keep in mind that $\lambda_4 = \lambda_{13} = \lambda_{14} = 2$.

In the finite version of the model, the price coefficients are now time-varying. At each of the steps described above, a conjectured price must be specified. To fix ideas, here are the prices at each trading period:

$$P_T = g_T\left(D_T, X_{T-3}\right) + a_{T,1}\varepsilon_{T+1}^d$$

$$
\begin{aligned}
&+\begin{pmatrix} b_{T,3} & b_{T,2} & b_{T,1} \end{pmatrix}\begin{pmatrix} \varepsilon^x_{T-2} & \varepsilon^x_{T-1} & \varepsilon^x_{T} \end{pmatrix}^\top \\
P_{T-1} =& g_{T-1}\left(D_{T-1}, X_{T-4}\right) + \begin{pmatrix} a_{T-1,2} & a_{T-1,1} \end{pmatrix}\begin{pmatrix} \varepsilon^d_{T} & \varepsilon^d_{T+1} \end{pmatrix}^\top \\
&+\begin{pmatrix} b_{T-1,3} & b_{T-1,2} & b_{T-1,1} \end{pmatrix}\begin{pmatrix} \varepsilon^x_{T-3} & \varepsilon^x_{T-2} & \varepsilon^x_{T-1} \end{pmatrix}^\top \\
&\ldots \\
P_{t} =& g_{t}\left(D_{t}, X_{t-3}\right) + \begin{pmatrix} a_{t,3} & a_{t,2} & a_{t,1} \end{pmatrix}\begin{pmatrix} \varepsilon^d_{t+1} & \varepsilon^d_{t+2} & \varepsilon^d_{t+3} \end{pmatrix}^\top \\
&+\begin{pmatrix} b_{t,3} & b_{t,2} & b_{t,1} \end{pmatrix}\begin{pmatrix} \varepsilon^x_{t-2} & \varepsilon^x_{t-1} & \varepsilon^x_{t} \end{pmatrix}^\top \\
&\ldots \\
P_{2} =& g_{2}\left(D_{2}, X_{0}\right) + \begin{pmatrix} a_{2,3} & a_{2,2} & a_{2,1} \end{pmatrix}\begin{pmatrix} \varepsilon^d_{3} & \varepsilon^d_{4} & \varepsilon^d_{5} \end{pmatrix}^\top \\
&+\begin{pmatrix} b_{2,2} & b_{2,1} \end{pmatrix}\begin{pmatrix} \varepsilon^x_{1} & \varepsilon^x_{2} \end{pmatrix}^\top \\
P_{1} =& g_{1}\left(D_{1}, X_{0}\right) + \begin{pmatrix} a_{1,3} & a_{1,2} & a_{1,1} \end{pmatrix}\begin{pmatrix} \varepsilon^d_{2} & \varepsilon^d_{3} & \varepsilon^d_{4} \end{pmatrix}^\top \\
&+ b_{1,1}\varepsilon^x_{1},
\end{aligned}
$$

where the function $g_t\left(D_{(\cdot)}, X_{(\cdot)}\right)$ is defined as

$$
g_t\left(D_{(\cdot)}, X_{(\cdot)}\right) = \bar{\alpha}_t \bar{D} + \alpha_t D_{(\cdot)} + \bar{\beta}_t \bar{X} + \beta_t X_{(\cdot)}
$$

The equations for the price coefficients are built recursively starting at time T. The unknown coefficients are $\bar{\alpha}_t$, α_t, $\bar{\beta}_t$, β_t, $a_{t,j}$, $b_{t,j}$, for $t = 1..T$ and $j = 1, 2, 3$. Once one arrives recursively at time $t = 1$, solve globally for the fixed point. The results of these computations are used to build Figure 8.

A.5 Pricing Kernel and the Short-Term Asset

To compute $P_{t,n}$ one has to start recursively at time $t + n - 1$:

$$
P_{t+n-1,1} = \hat{\mathbb{E}}_t \left[M_{t+n} D_{t+n}\right],
$$

where $\hat{\mathbb{E}}_t$ is obtained by averaging the individual expectations across agents. The individual pricing kernels are

$$
M^i_{t+n} = \frac{e^{-\gamma \tilde{w}^i_{t+n}}}{R\mathbb{E}^i_{t+n-1}[e^{-\gamma \tilde{w}^i_{t+n}}]} = \frac{e^{-\gamma x^i_{t+n-1}(P_{t+n}+D_{t+n})}}{R\mathbb{E}^i_{t+n-1}[e^{-\gamma x^i_{t+n-1}(P_{t+n}+D_{t+n})}]}.
$$

It follows that

$$
\begin{aligned}
&\mathbb{E}^i_{t+n-1}[M^i_{t+n} D_{t+n}] = \\
&\qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} M^i_{t+n-1} D_{t+n} \mathcal{N}(P_{t+n} + D_{t+n}, D_{t+n}) d(P_{t+n} + D_{t+n}) dD_{t+n},
\end{aligned}
$$

where $\mathcal{N}(\cdot, \cdot)$ represents the bivariate normal distribution.

The above double integral can be computed in closed form by means of the Gaussian assumption. The result is a normally distributed variable. Then, go one step back to $\mathbb{E}^i_{t+n-2}[M^i_{t+n-1} P_{t+n-1,1}]$. Apply similar calculations in this case to obtain $P_{t+n-2,2}$. The iterations are repeated until one obtains $P_{t,n}$.

A.6 Trading Volume

There are an infinity of types of investor, depending on the couple of signals $\{n^i_{t,1}, n^i_{t,2}\}$. Let us index these types by m. The trading volume of investors of type m is (ignoring noise trading)

$$V_t^m = \frac{1}{2}\int_{i\in m} \left| x_t^i - x_{t-1}^i \right| di, \tag{A.9}$$

where x_t^i is their optimal demand, defined in (4).

For agents of type $\{n^i_{t,1}, n^i_{t,2}\}$, the optimal demand can be written

$$\begin{aligned} x_t^i = & \frac{f(\bar{D}, D_t, \bar{X}, X_{t-3}) - R(\bar{\alpha}\bar{D} + \alpha D_t + \bar{\beta}\bar{X} + \beta X_{t-3})}{\gamma \mathrm{Var}_t^i[P_{t+1} + D_{t+1}]} \\ & + \frac{\psi \mathbb{M}^i + (b^* \mathbb{B}^{-1}\ 0\ 0\ 0) - (R\ 0\ 0\ 0\ 0\ 0)}{\gamma \mathrm{Var}_t^i[P_{t+1} + D_{t+1}]} \begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix}. \end{aligned} \tag{A.10}$$

By the market clearing condition (5), the first term in (A.10) is equal to $\bar{X}/(\bar{K}_t \mathrm{Var}_t^i[P_{t+1} + D_{t+1}])$.

I assume that the type is time invariant, in the sense that the investors remain of the same type for two successive generations. This assumption makes easier the computations (see Watanabe, 2008, for a similar assumption). Separate calculations without this assumption show that results do not change qualitatively. It follows that the trading strategy of investor i is

$$x_t^i - x_{t-1}^i = \mathbb{Q}^i \left[\begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix} - \begin{pmatrix} \mathbb{P}_{t-1} \\ v_{t-1}^i \\ W_{t-1}^i \end{pmatrix} \right],$$

where $\mathbb{Q}^i \equiv \frac{\psi \mathbb{M}^i + (b^* \mathbb{B}^{-1}\ 0\ 0\ 0) - (R\ 0\ 0\ 0\ 0\ 0)}{\gamma \mathrm{Var}_t^i[P_{t+1} + D_{t+1}]}$. Note that $\mathbb{Q}^i$ is a vector of dimension 1×6. Denote by $\mathbb{Q}^i_{4-6}$ the vector of dimension 1×3 which contains the 3 last elements of $\mathbb{Q}^i$, and by $\mathbb{Q}^i_j$ the jth element of $\mathbb{Q}^i$. After some manipulations, one obtains

$$\mathbb{Q}^i \begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix} = \mathbb{Q}^i \left[\mathbb{H}\epsilon_t^d + \begin{pmatrix} \mathbb{B} \\ \mathbf{0}_{3\times 3} \end{pmatrix} \epsilon_t^x \right] + \mathbb{Q}^i_{4-6} \begin{pmatrix} \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix}.$$

In a similar way

$$\mathbb{Q}^i \begin{pmatrix} \mathbb{P}_{t-1} \\ v_{t-1}^i \\ W_{t-1}^i \end{pmatrix} = \mathbb{Q}^i \left[\mathbb{H}\epsilon_{t-1}^d + \begin{pmatrix} \mathbb{B} \\ \mathbf{0}_{3\times 3} \end{pmatrix} \epsilon_{t-1}^x \right] + \mathbb{Q}^i_{4-6} \begin{pmatrix} \varepsilon_{t-1}^{vi} \\ \varepsilon_{t-1,2}^{wi} \\ \varepsilon_{t-1,1}^{wi} \end{pmatrix}.$$

The optimal trading strategy becomes

$$
\begin{aligned}
x_t^i - x_{t-1}^i &= \mathbb{Q}^i \left[\begin{pmatrix} \mathbf{0}_{6\times1} & \mathbb{H} \end{pmatrix} - \begin{pmatrix} \mathbb{H} & \mathbf{0}_{6\times1} \end{pmatrix} \right] \begin{pmatrix} \varepsilon_t^d \\ \epsilon_t^d \end{pmatrix} \\
&+ \mathbb{Q}^i \left[\begin{pmatrix} \mathbf{0}_{3\times1} & \mathbb{B} \\ \mathbf{0}_{3\times1} & \mathbf{0}_{3\times3} \end{pmatrix} - \begin{pmatrix} \mathbb{B} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times1} \end{pmatrix} \right] \begin{pmatrix} \varepsilon_{t-3}^x \\ \epsilon_t^x \end{pmatrix} \\
&+ \mathbb{Q}_{4-6}^i \left[\begin{pmatrix} \varepsilon_{t-1}^{vi} \\ \varepsilon_{t-1,2}^{wi} \\ \varepsilon_{t-1,1}^{wi} \end{pmatrix} - \begin{pmatrix} \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix} \right]
\end{aligned} \tag{A.11}
$$

Formula (A.9) requires the computation of the cross-sectional average across investors of the absolute value of the normal variable stated in (A.11). The absolute value of a normal variable follows a Folded normal distribution. The mean of this variable is equal to the first two lines in (A.11), while the variance of this variable is equal to

$$
\mathrm{Var}(x_t^i - x_{t-1}^i) = 2\mathbb{Q}_{4-6}^i \begin{pmatrix} \sigma_v^2 & 0 & 0 \\ 0 & \frac{\sigma_v^2}{n_{t,2}^i} & 0 \\ 0 & 0 & \frac{\sigma_v^2}{n_{t,1}^i} \end{pmatrix} \mathbb{Q}_{4-6}^{i\top} - 2\mathbb{Q}_5^i \mathbb{Q}_4^i \frac{\sigma_v^2}{n_{t,2}^i} - 2\mathbb{Q}_6^i \mathbb{Q}_5^i \frac{\sigma_v^2}{n_{t,1}^i}. \tag{A.12}
$$

For the computation of this variance one recognizes that $\varepsilon_{t-1,2}^{wi}$ and ε_t^{vi} are correlated. Same for $\varepsilon_{t-1,1}^{wi}$ and $\varepsilon_{t,2}^{wi}$. The last two terms in (A.12) reflect this fact.

The Folded normal distribution formula can be applied at this point. The trading volume of type $\{n_{t,1}^i, n_{t,2}^i\}$ investors at time t, V_t^m, becomes a function of both dividend and supply innovations, i.e. ε_t^d to ε_{t+3}^d and ε_{t-3}^x to ε_t^x. The expected value of the trading volume can then be computed by very fast simulations. The total expected trading volume in the economy can then be computed with the summation

$$
V_t = \sum_{n_{t,1}^i=1}^{\infty} \sum_{n_{t,2}^i=1}^{\infty} \mu\left(n_{t,1}^i, n_{t,2}^i\right) V_t^m + \frac{1}{2}\left|X_t - X_{t-1}\right|
$$

The left panel of Figure 10 is a result of these computations for different values of λ.

For the dynamics of the trading volume, one has to go back to (A.10). In the case of an economy where all the agents are of the same type, the optimal demand becomes

$$
x_t^i = \bar{X} + \frac{\psi\mathbb{M} + (b^*\mathbb{B}^{-1}\ 0\ 0\ 0) - (R\ 0\ 0\ 0\ 0\ 0)}{\gamma \mathrm{Var}_t[P_{t+1} + D_{t+1}]} \begin{pmatrix} \mathbb{P}_t \\ v_t^i \\ W_t^i \end{pmatrix}.
$$

Note that $\mathbb{M}$ and $\mathrm{Var}_t[P_{t+1} + D_{t+1}]$ are not indexed by i since they are identical across agents. However, one has to take into account that the price coefficients are changing now, depending on the level of λ. At each point in time, the optimal trading strategy $x_t^i - x_{t-1}^i$ is computed by taking care of these changes. Then, the trading volume is computed by using the same technique as above. The results of these computations are used to build the right panel of Figure 10.

In order to analyze investors' trading strategies associated with differential information (to build Figure 13), I compute the aggregate trading volume for each group of investors (after adding the noise trades, to isolate the informational demand):

$$\Delta x_t^i - \Delta X_t = x_t^i - x_{t-1}^i - \left(\varepsilon_t^x - \varepsilon_{t-1}^x\right).$$

I use directly $\kappa_x = 0$, as in the benchmark calibration, to keep things simpler. By using (A.11):

$$\begin{aligned}\Delta x_t^i - \Delta X_t = \mathbb{Q}^i &\left[\begin{pmatrix} \mathbf{0}_{6\times 1} & \mathbb{H} \end{pmatrix} - \begin{pmatrix} \mathbb{H} & \mathbf{0}_{6\times 1} \end{pmatrix}\right] \begin{pmatrix} \varepsilon_t^d \\ \epsilon_t^d \end{pmatrix} \\ &+ \left[\mathbb{Q}^i \left[\begin{pmatrix} \mathbf{0}_{3\times 1} & \mathbb{B} \\ \mathbf{0}_{3\times 1} & \mathbf{0}_{3\times 3} \end{pmatrix} - \begin{pmatrix} \mathbb{B} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 1} \end{pmatrix}\right] + \begin{pmatrix} 0 & 0 & 1 & -1 \end{pmatrix}\right] \begin{pmatrix} \varepsilon_{t-3}^x \\ \epsilon_t^x \end{pmatrix} \\ &+ \mathbb{Q}_{4-6}^i \left[\begin{pmatrix} \varepsilon_{t-1}^{vi} \\ \varepsilon_{t-1,2}^{wi} \\ \varepsilon_{t-1,1}^{wi} \end{pmatrix} - \begin{pmatrix} \varepsilon_t^{vi} \\ \varepsilon_{t,2}^{wi} \\ \varepsilon_{t,1}^{wi} \end{pmatrix}\right]\end{aligned}$$

The informational trading strategy is a linear function of dividend and supply innovations. The correlation with the price difference ΔP_t can then be computed in closed form. The results of these computations are used to build Figure 13.

A.7 Infinite-Horizon Investors

I describe here the method for solving the equilibrium price in the model with infinite-horizon investors. Asset demand is more complex than in the two-period case. In the infinite-horizon case the maximization problem is

$$U_t^i = \max_{\left(c_{t+s}^i, x_{t+s}^i\right)_{s\geq 0}} -\mathbb{E}_t^i \left[\sum_{s=0}^{\infty} \rho^s e^{-\gamma c_{t+s}^i}\right], \tag{A.13}$$

with $\rho \in (0,1)$ and subject to the intertemporal budget constraint

$$w_{t+1}^i = (w_t^i - c_t^i)R + x_t^i(P_{t+1} + D_{t+1} - RP_t).$$

When $T > 1$ (e.g., when the private information is long-lived) investors use information from previous periods to update their expectations. This leads to the infinite regress problem (see Bacchetta and Wincoop, 2008, for a clear treatment of this topic). The problem arises in both the overlapping generations or the infinite-horizon cases. The equilibrium price can be written as

$$P_t = \bar{\alpha}^* \bar{D} + \bar{\beta}^* \bar{X} + A(L)\varepsilon_t^d + B(L)\varepsilon_t^x,$$

where $A(L) = a_1 + a_2 L + ...$ and $B(L) = b_1 + b_2 L + ...$ are infinite order polynomials in the lag operator L. Although this makes the price dependent on the infinite state space, in the overlapping generations case it is easily verified that D_t and X_{t-T} collect all the past shocks ε_j^d for $j \leq t$ and ε_j^x for $j \leq t - T$ respectively. The fixed point problem to be solved becomes finite dimensional, and the price takes the conjectured form (8).

In the infinite-horizon case, the portfolio maximization problem is substantially more complicated. Investors take into account uncertainty about future expected returns and form dynamic hedging demands, which might be relevant in a model with long-lived information as the present one. The hedge term depends on the infinite state space. This complicates matters, because the fixed point problem to be solved cannot be reduced to a finite dimensional one. However, an approximation to a desired accuracy level can be achieved by truncating the state space for sufficiently long lags. Bacchetta and Wincoop (2006) show how to do that in a previous version of the paper. I adopt this method here.

The Bellman Optimality principle says that

$$U_t^i = \max_{c_t^i, x_t^i} \left[-e^{-\gamma c_t^i} + \rho \mathbb{E}_t^i U_{t+1}^i \right].$$

I compute the equilibrium price by assuming that the vector of observables is $Y_t^i = \left(\bar{D}\ D_t\ \bar{X}\ X_{t-3}\ \mathbb{p}_t^\top\ v_t^i\ W_t^{i\top} \right)^\top$. This allows me to keep the conjectured form of the price as in (8). This vector of observables can be extended by adding lags. A proper modification of the price conjecture must be performed in that case.

I conjecture that the value function is:

$$U_t^i = -\alpha_1 \exp\left\{ -\alpha_2 w_t^i - \frac{1}{2} Y_t^{i\top} V Y_t^i \right\}, \tag{A.14}$$

where α_1 and α_2 are scalars and V is a square matrix to be determined in equilibrium.

By use of the projection theorem, $P_{t+1} + D_{t+1} - RP_t$ and Y_{t+1}^i can be written as follows:

$$\begin{aligned} P_{t+1} + D_{t+1} - RP_t &= \Theta^\top Y_t^i + \Theta_3^\top \epsilon_t^i \\ Y_{t+1}^i &= N_1 Y_t^i + N_2 \epsilon_t^i \end{aligned}$$

where ϵ_t^i is the vector of shocks defined as

$$\epsilon_t^i = \left(\varepsilon_{t+4}^d,\ \varepsilon_{t+1}^x,\ \epsilon_t^{d\top} - \mathbb{E}_t^i \epsilon_t^{d\top},\ \epsilon_t^{x\top} - \mathbb{E}_t^i \epsilon_t^{x\top},\ \varepsilon_t^{vi},\ \varepsilon_{t,2}^{wi},\ \varepsilon_{t,1}^{wi},\ \varepsilon_{t+1}^{vi},\ \varepsilon_{t+1,2}^{zi},\ \varepsilon_{t+1,1}^{zi} \right).$$

All these shocks have been defined in Section 3, except $\varepsilon_{t+1,2}^{zi}$ and $\varepsilon_{t+1,1}^{zi}$. They represent the innovations in the incremental signals gathered by investor i between time t and $t+1$ about D_{t+3} and D_{t+2} respectively. Denote the covariance matrix of these shocks by Σ.

$P_{t+1} + D_{t+1} - RP_t$ and Y_{t+1}^i can be replaced in the conjectured value function (A.14). Then, $\mathbb{E}_t^i U_{t+1}^i$ becomes

$$\begin{aligned} \mathbb{E}_t^i U_{t+1}^i = -\alpha_1 \exp\Big\{ & -\alpha_2 \left(w_t^i - c_t^i \right) R - \alpha_2 x_t^i \Theta^\top Y_t^i - \alpha_2 x_t^i \Theta_3^\top \epsilon_{t+1}^i \\ & - Y_t^{i\top} N_1^\top V N_2 \epsilon_{t+1}^i - \frac{1}{2} \epsilon_{t+1}^{i\top} N_2^\top V N_2 \epsilon_{t+1}^i \Big\}. \end{aligned}$$

The following standard lemma in multivariate normal calculus is necessary:

Lemma 1. *Let ε be a multivariate normal random variable, with zero mean and covariance matrix Σ. Let b be a constant vector, and B a constant symmetric semi-positive definite matrix. Define $\Omega = (B + \Sigma^{-1})^{-1}$. Then*

$$\mathbb{E} \exp\left\{ -b^\top \varepsilon - \frac{1}{2} \varepsilon^\top B \varepsilon \right\} = \frac{1}{|\Sigma|^{1/2} |\Omega|^{-1/2}} \exp\left\{ \frac{1}{2} b^\top \Omega b \right\},$$

where $|X|$ denotes the determinant of the square matrix X.

In my case, Ω is defined as $\left(\Sigma^{-1} + N_2^\top V N_2 \right)$. Use this lemma to transform $\mathbb{E}_t^i U_{t+1}^i$, and then write the first order condition with respect to x_t^i. After a few manipulations one finds

$$x_t^i = \frac{\Theta^\top Y_t^i - \Theta_3^\top \Omega N_2^\top V^\top N_1 Y_t^i}{\alpha_2 \Theta_3^\top \Omega \Theta_3}.$$

The first term in the numerator represent the expected return, as in the overlapping generations case. The second term represent the hedge against expected return changes.

Replace the optimal demand in (A.13) to obtain

$$U_t^i = -e^{-\gamma c_t^i} - \frac{\rho\alpha_1}{|\Sigma|^{1/2}|\Omega|^{-1/2}} \exp\left\{-\alpha_2\left(w_t^i - c_t^i\right)R - \frac{1}{2}Y_t^{i\top}N_1^\top V N_1 Y_t^i \right.$$
$$\left. -\frac{1}{2}\alpha_2^2\left(x_t^i\right)^2\Theta_3^\top\Omega\Theta_3 + \frac{1}{2}Y_t^{i\top}N_1^\top V N_2\Omega N_2^\top V^\top N_1 Y_t^i\right\}.$$

Write the first order condition for consumption and then replace in the above equation. By verifying the guess of the value function (A.14) one obtains α_1, α_2, and the following implicit equation for V:

$$V = \frac{1}{R}\left(\frac{\left(\Theta - N_1^\top V N_2\Omega^\top\Theta_3\right)\left(\Theta^\top - \Theta_3^\top\Omega N_2^\top V N_1\right)}{\Theta_3^\top\Omega\Theta_3} + N_1^\top V N_1 \right.$$
$$\left. - N_1^\top V N_2\Omega N_2^\top V^\top N_1\right). \tag{A.15}$$

The parameter α_2 is equal to $\gamma(R-1)/R$ (this is true in all portfolio problems for CARA agents).

Thus, the market clearing condition plus the verification that the cojecture of the value function is correct solves for all the parameters of the model. The numerical procedure is as follows:

1. Start with a given V, usually $V = 0$.
2. Solve for the price coefficients using the market clearing condition. If the numerical solver takes too long, there is a very fast alternative. Assume starting values for the parameters. Solve the equilibrium price equation and map the assumed parameters in new values. Continue the process until it converges. This is usually the case because of the fixed point nature of the problem.
3. Once the price coefficients are found, verify if V satisfy equation (A.15). This is done by an iteration technique. A new value of V is found at this step.
4. Use this new value of V at point 1 and repeat the algorithm until convergence.

As in Albuquerque and Miao (2010) and Bacchetta and Wincoop (2006), the results for the infinite horizon model are very close to those for the overlapping generations model. Table 3 shows this.

Model / Coeff.	$\bar{\alpha}$	α	β	β	a_1	a_2	a_3	b_1	b_2	b_3
Overlapp. gen.	249.9	0.15	-153.3	0	0.0004	0.0159	0.2092	-1.1709	-0.0402	-0.0011
Infinite hor.	249.9	0.15	-147.2	0	0.0025	0.0281	0.2134	-1.2104	-0.0794	-0.0060

Table 3: Price coefficients in overlapping generations and infinite-horizon.