Trade Costs, Heterogeneous Firms and International Portfolio Choice

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Abstract

This paper builds on a two-country dynamic stochastic general equilibrium (DSGE) model in which households can invest in home and foreign equities. First, a benchmark model is created to obtain several previous results of the existing literature. A closed form solution is derived for the optimal portfolio holdings. In this benchmark model it is shown why the hedging of both real exchange rate risk and labor income risk will counterfactually generate foreign equity bias. Then, new features are introduced, such as non-traded goods, heterogeneous firms and fixed export costs. The interactions between these new features helps to explain the home equity bias documented in many empirical studies.

JEL classification: F30, International finance - general; G11, Portfolio choice; G15, International financial markets

Keywords: International portfolio diversification; Home asset bias; Home asset preference; Portfolio choice

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1 Introduction

Why residents of most major industrialized countries hold most of their wealth in domestic assets, forgoing the benefits of diversifying their portfolios? Even after recent research have documented an explosion of international asset trade, the so called *home equity bias* is still sizable[1]. Sercu and Vanpée (2007) is one of the latest studies documenting this fact and making a complete review of the recent literature on equity home bias.

The goal of this paper is twofold. First, I introduce into a DSGE model portfolio choice in equities by using the method developped by Devereux and Sutherland (2007). This method can be implemented both when markets are complete and incomplete. Second, the paper shows that for reasonable parameter values it is efficient for home agents to hold a portfolio biased toward home equity. Moreover, the results are reinforced by the fact that in the paper, I abstract from barriers to international capital movements and assume that any investor can purchase any security without transaction costs.

I start by building a dynamic stochastic general equilibrium model of international portfolio choice, with home bias in consumption generated by both a preference parameter and trade costs in goods, à la Coeurdacier (2006). In this setup, I use the method developped by Devereux and Sutherland (2007) to solve for the optimal portfolios.

With this benchmark model at hand, I am able to replicate several results from the existing literature and to explain the difficulties that previous studies had to create home equity bias. Precisely, the benchmark model is able to generalize the results of Lucas (1982), Baxter and Jermann (1997), Cole and Obstfeld (1991), Coeurdacier (2006), and Kollmann (2006).

The analysis uses a two-country general equilibrium model with tradable goods. The production sector of each country is subject to stochastic productivity shocks. Households supply labor elastically and maximize expected inter-temporal utility from consumption. They finance their consumption expenditures by trading in equities issued by firms in both countries.

Then, I deviate from this benchmark model by inserting heterogeneity of firms (in terms of productivity), fixed export costs and a non-tradable sector, as in Ghironi and Melitz (2005). The motivation for fixed export costs comes from a relatively recent and growing area of interest in the economic literature (Das et al. (2007), Jensen and Bernard (2001), Melitz (2003)). These fixed export costs, when in interaction with the heterogeneous firms setup and with the non-tradable sector, are able to generate home equity preference.

Moreover, I extend the model by adding more exogenous shocks than assets, in a similar way as in Coeurdacier et al. (2007). However, the shocks in this paper are much more intuitive and come from a more structural setup. By introducing other than just productivity shocks, I relax the assumption that markets are complete, thus the optimal portfolios are studied in a general equilibrium framework with incomplete asset markets. Finding an equilibrium of a model with incomplete

[1] see Lane and Milesi-Ferretti (2001), Milesi-Ferretti and Lane (2005), Lane and Milesi-Ferretti (2007) and Gourinchas and Rey (2007).

markets is challenging, because in such economy any shifts in the distribution of wealth affect the dynamics of asset returns, which in turn determine the variations in risk-premia and investors' protfolios. Therefore, I use the solution method developped by Devereux and Sutherland (2007) to solve for optimal portfolios even in incomplete markets.

The results will show significant improvements, not only in terms of home equity positions, but because markets are now incomplete, this helps explaining the consumption-real exchange rate anomaly (see Backus and Smith (1993)). The results will also be more robust to parameter changes.

The remainder of the paper is organized as follows. Section 2 makes a brief review of the related literature. In section 3 I build the benchmark model. Section 4 provides a brief description of the solution method developped by Devereux and Sutherland (2007). Section 5 shows the closed form solution and discuss the results for the benchmark model. In section 6 I build a new setup with a non-tradable sector, fixed export costs and heterogeneous firms. A closed form solution is provided, then I make the extension to an incomplete markets setup and discuss the findings. Section 7 concludes.

2 Literature Review

A large strand of literature in international portfolio choice has tried to explain the equity home bias. Some authors study portfolio choice in models with consumption home bias[2]. This comes in line with Obstfeld and Rogoff (2006) who have argued that the trade costs (which generate consumption home bias) can solve the equity home bias puzzle. Kollmann (2006) generates portfolio home bias in an endowment economy with home bias in consumption. Coeurdacier (2006) introduce a combination of small frictions and trade costs and finds that the larger home bias in consumption, the larger the home bias in portfolios. But the majority of these models can only generate equity home bias when the substitution elasticity between domestic and imported goods is smaller than unity. Recent literature (see Coeurdacier et al. (2007)) has argued that a model with just supply shocks cannot generate equity home bias, except in the case when relative equity returns is highly positively correlated with terms of trade. Empirically, this correlation is close to zero[3].

The Coeurdacier et al. (2007) study is the most closely related to my work. However, there are two main differences between my work and Coeurdacier et al. (2007). First, I show how in my model a productivity shock can propagate into the economy both as a supply and a redistributive shock (a shock that increases dividends of domestic firms while reducing domestic labor income). By constrast, in their setup they introduce both supply and redistributive shocks separately. Second, my setup offers a way to introduce demand shocks through the variety effects, in a much more intuitive way that in Coeurdacier et al. (2007).

Other studies analyse the impact of non-tradable income on equity home bias.

[2] see Coeurdacier (2006), Kollmann (2006), Hnatkovska (2005).

[3] van Wincoop and Warnock (2006)

According to this literature, the presence of labor income either worsens the home equity bias (see Baxter and Jermann (1997)) or helps explaining it (see Bottazzi et al. (1996), Julliard (2003), and Engel and Matsumoto (2006)).

This paper is also related to the literature on the role of relative prices in international risk-sharing. Cole and Obstfeld (1991) showed in a two-country endowment economy with complete markets that, when preferences are symmetric Cobb-Douglas or separable, any variations in relative endowments induces an exactly off-setting change in relative prices, and thus any portfolio ensures perfect risk sharing. Heathcote and Perri (2007) goes into the same direction by extending the Cole and Obstfeld (1991) analysis to a production economy. My study is also related to Uppal (1993), who has shown in a complete general equilibrium setting with shipping costs that only investors less risk-averse than log will prefer home stocks.

Recently, new methodologies have been developped to solve for optimal portfolios in DSGE models, whether markets are complete or incomplete. Devereux and Sutherland (2007) build one of this methods, which is the simplest since it complements standard quantitative methods commonly used in DSGE models. Related and similar works are Tille and van Wincoop (2007) and Evans and Hnatkovska (2005).

3 A Benchmark Model

3.1 Households and Firms

The world consists of two symmetric countries, home and foreign (foreign variables are denoted by an asterisk). Each country is populated by a unit mass of atomistic households, which maximize expected intertemporal utility from consumption and supply labor elastically. The utility of the home agents is

$$U_t = \mathbb{E}_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(\frac{C_\tau^{1-\rho}}{1-\rho} - \kappa \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right) \right]$$

where φ is the Frisch elasticity of labor supply to wages (intertemporal elasticity of substitution in labor supply) and $\kappa > 0$. The subjective discount factor is $\beta \in (0,1)$ and $\rho > 0$ is the coefficient of relative risk aversion. The home composite consumption aggregate over home and foreign good categories, C_t, is defined as

$$C_t = \left[\mu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\mu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

with $\mu \geq \frac{1}{2}$ the parameter of assymetry in preferences and θ the elasticity of substitution between home and foreign goods. All goods are tradable, but if goods are shipped to the another country, then a fraction $\tau < 1$ is lost in transit, as in Coeurdacier (2006). $C_{H,t}$ and $C_{F,t}$ are CES aggregators over varieties produced in the home (foreign) country:

$$C_{H,t} = \left(\int^{N_D} c_t(h)^{\frac{\phi-1}{\phi}} dh \right)^{\frac{\phi}{\phi-1}}, \quad C_{F,t} = \left(\int^{N_D^*} c_t(f)^{\frac{\phi-1}{\phi}} df \right)^{\frac{\phi}{\phi-1}}$$

where $\phi > 1$ is the symmetric elasticity of substitution across goods, which is assumed to be the same for home and for foreign country. The aggregate consumer price index for home agents is therefore

$$P_t = \left[\mu P_{H,t}^{1-\theta} + (1-\mu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

with

$$\begin{aligned} P_{H,t} &= \left[\int^{N_D} p_t(h)^{1-\phi} dh\right]^{\frac{1}{1-\phi}} = N_D^{\frac{1}{1-\phi}} p_t(h) \\ P_{F,t} &= \left[\int^{N_D^*} p_t(f)^{1-\phi} df\right]^{\frac{1}{1-\phi}} = (N_D^*)^{\frac{1}{1-\phi}} p_t(f) \end{aligned}$$

Households in each country hold two types of assets: shares in a mutual fund of domestic firms and shares in a mutual fund of foreign firms. There is frictionless international trade in equities. The home budget constraint is given by[4]

$$W_t = \alpha_{E,t-1} r_{E,t} + \alpha_{E^*,t-1} r_{E,t}^* + w_t L_t + \Pi_t - C_t$$

where W_t is the real net wealth[5], w_t is the real wage rate, Π_t is the total real profit of home firms, α_E and α_{E^*} are holdings of home and foreign equities. The real return of home and foreign equities are respectively r_E and r_E^*. All the financial variables $\left(W_t, W_t^*, \alpha_{E,t}, \alpha_{E^*,t}, \alpha_{E,t}^*, \alpha_{E^*,t}^*, r_{E,t}, r_{E,t}^*\right)$ are defined in terms of the home consumption good.

There is a fixed number of firms in each country, N_D and N_D^*. Each firm is producing a different variety. Firms are homogeneous in the benchmark model. The extension to a case with heterogeneous firms and only a fraction of exporting firms is done in Section 6. Each firm produce a differentiated variety with an homogeneous technology which requires only labor:

$$Y_t(h) = A_t l_t(h)$$

All firms face a residual demand curve with constant elasticity ϕ in both markets, and set fully flexible prices that reflect the same proportional markup $\phi/(\phi-1)$ over marginal cost. Let $p_t(h)$ and $p_t^*(h)$ denote nominal domestic and export prices of a home firm. The domestic and export prices in real terms relative to the price index in the destination market are given by

$$\frac{p_t(h)}{P_t} = \frac{\phi}{\phi-1} \frac{w_t}{A_t} \tag{1}$$

[4] I am assuming that, by default, all capital in a country is owned by the residents of that country. This allows me to treat equity claims to capital income as inside assets, i.e. assets in zero net supply, and is purely an accounting convention. This approach makes the derivations easier and prepares the setup to the solution method of Devereux and Sutherland (2007).

[5] $W_t = \alpha_{1,t} + \alpha_{2,t}$ represent also the total net claims of home agents on the foreign country at the end of period t (i.e. the net foreign assets of home agents).

$$\frac{p_t^*(h)}{P_t^*} = \frac{1}{Q_t}\frac{1}{1-\tau}\frac{p_t(h)}{P_t} = \frac{1}{Q_t}t\frac{\phi}{\phi-1}\frac{w_t}{A_t}$$

where $Q_t = P_t^*/P_t$ is the consumption-based real exchange rate (units of home consumption per unit of foreign consumption) and $t = \frac{1}{1-\tau}$. The domestic price is expressed in units of home consumption. The export price is expressed in units of foreign consumption.

Overall firm productivity is subject to aggregate (country-specific) shocks. Home and foreign productivity shocks are auto-regressive processes of the form

$$\log A_t = \zeta_A \log A_{t-1} + \varepsilon_{A,t}, \quad \log A_t^* = \zeta_A^* \log A_{t-1}^* + \varepsilon_{A,t}^*$$

where $\varepsilon_{A,t}$ and $\varepsilon_{A,t}^*$ are zero-mean i.i.d. shocks with $Var\left[\varepsilon_{A,t}\right] = Var\left[\varepsilon_{A,t}^*\right] = \sigma_A^2$ and $Cov\left[\varepsilon_{A,t}, \varepsilon_{A,t}^*\right] = 0$. The covariance is set to zero in the benchmark model, but the Devereux and Sutherland (2007) methodology allows for changes. In the benchmark model I will also assume $\zeta_A = \zeta_A^*$.

3.2 Equilibrium

3.2.1 Households and Firms

In what follows I will use the foreign equity as the numeraire. This allows me to re-write the home budget constraint as

$$W_t = \alpha_{E,t-1} r_{x,t} + W_{t-1} r_{E,t}^* + w_t L_t + \Pi_t - C_t$$

where $r_{x,t} = r_{E,t} - r_{E,t}^*$ measures the excess return of home equity on foreign equity. A similar equation is obtained for the foreign budget constraint

$$\frac{1}{Q_t} W_t^* = \frac{1}{Q_t}\left[\alpha_{E,t-1}^* r_{x,t} + W_{t-1}^* r_{E,t}^*\right] + w_t^* L_t^* + \Pi_t^* - C_t^* \tag{2}$$

Note that the real exchange rate Q_t enters (2) because wealth, portfolio holdings, and returns are defined in terms of the home consumption good.

At the end of each period, agents select the portfolio of equities to hold in the following period. Thus, the first order conditions with respect to $\alpha_{E,t-1}$ (for the home agent) and $\alpha_{E,t-1}^*$ (for the foreign agent) are respectively

$$C_t^{-\rho} = \beta \mathbb{E}_t\left[C_{t+1}^{-\rho} r_{E,t+1}^*\right], \quad (C_t^*)^{-\rho} = \beta \mathbb{E}_t\left[\frac{Q_t}{Q_{t+1}}\left(C_{t+1}^*\right)^{-\rho} r_{E,t+1}^*\right]$$

$$\begin{aligned} \mathbb{E}_t\left[C_{t+1}^{-\rho} r_{E,t+1}\right] &= \mathbb{E}_t\left[C_{t+1}^{-\rho} r_{E,t+1}^*\right] && (3) \\ \mathbb{E}_t\left[\frac{1}{Q_{t+1}}\left(C_{t+1}^*\right)^{-\rho} r_{E,t+1}\right] &= \mathbb{E}_t\left[\frac{1}{Q_{t+1}}\left(C_{t+1}^*\right)^{-\rho} r_{E,t+1}^*\right] && (4) \end{aligned}$$

The optimal consumption-leisure tradeoff implies (for home and foreign agents respectively)

$$w_t = \kappa C_t^{\rho} L_t^{\frac{1}{\varphi}}, \quad w_t^* = \kappa \left(C_t^*\right)^{\rho} \left(L_t^*\right)^{\frac{1}{\varphi}}$$

The home consumer's demand for home and foreign composite goods may be written as

$$C_{H,t} = \mu \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t, \quad C_{F,t} = (1-\mu) \left(\frac{P_{F,t}}{P_t}\right)^{-\theta} C_t$$

The home consumer's demand for home and foreign varieties may be written as

$$c_t(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\phi} C_{H,t}, \quad c_t(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\phi} C_{F,t}$$

Firm profits are

$$\pi_t(h) = \frac{p_t(h)}{P_t} Y_t(h) - w_t l_t(h)$$

By using the domestic prices equation (1), we can express firm profits in terms of firm's revenue

$$\pi_t(h) = \frac{1}{\phi} \frac{p_t(h)}{P_t} Y_t(h)$$

Thus, we can write the aggregate profits as a constant fraction of total revenue

$$\Pi_t = N_D \pi_t(h) = \frac{1}{\phi} N_D \frac{p_t(h)}{P_t} Y_t(h)$$

3.2.2 Market Clearing

A firm satisfies two sources of demand: those of home and foreign households. The market clearing condition for the domestic firms goods in the domestic market is

$$\begin{aligned} Y_{D,t}(h) &= \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\phi} C_{H,t} = \mu \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\phi} \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t \\ &= \mu \left(\frac{p_t(h)}{N_D^{\frac{1}{1-\phi}} p_t(h)}\right)^{-\phi} \left(\frac{N_D^{\frac{1}{1-\phi}} p_t(h)}{P_t}\right)^{-\theta} C_t = \mu N_D^{\frac{\phi-\theta}{1-\phi}} \left(\frac{p_t(h)}{P_t}\right)^{-\theta} C_t \end{aligned}$$

A similar equation is found for the domestic firms in the foreign market (note that here we need to take into account the fact that when exporting, a fraction τ is lost in transit):

$$
\begin{aligned}
Y_{X,t}(h) &= \left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\phi} C_{H,t}^* = (1-\mu)\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\phi}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta}\frac{1}{t}C_t^* \\
&= (1-\mu)\, N_D^{\frac{\phi-\theta}{1-\phi}}\left(\frac{p_t^*(h)}{P_t^*}\right)^{-\theta} tC_t^*
\end{aligned}
$$

The total production of firms is thus $Y_t(h) = Y_{D,t}(h) + Y_{X,t}(h)$. Similar equations are obtained for foreign firms.

Labor market clearing implies

$$N_D l_t(h) = L_t$$

and a similar equation for the foreign country. Finally, financial assets are assumed to be in zero net supply, so financial markets in equilibrium must fulfill:

$$\alpha_{E,t} + \alpha_{E,t}^* = 0, \quad \alpha_{E^*,t} + \alpha_{E^*,t}^* = 0$$

In what follows, I will compute $\alpha_{E,t}$ using the methodology developped by Devereux and Sutherland (2007). It should be understood, therefore, that $\alpha_{E,t}^* = -\alpha_{E,t}$, $\alpha_{E^*,t} = W_t - \alpha_{E,t}$, and $\alpha_{E^*,t}^* = W_t^* + \alpha_{E,t}$. Furthermore, to provide an economic interpretation of the solution, I will re-express $\alpha_{E,t}$ in terms of the proportion of home equity held by home residents. The total value of home equity is $N_D v_t$, where v_t is the value of a home firm. Then the proportion held by home residents will be

$$\alpha_t^E = \frac{\alpha_{E,t} + N_D v_t}{N_D v_t}.$$

Returns of home and foreign firms are defined as follows

$$r_{E,t} = \frac{\pi_t + v_t}{v_{t-1}}, \quad r_{E,t}^* = \frac{\pi_t^* + v_t^*}{v_{t-1}^*}\frac{Q_t}{Q_{t-1}}$$

Note that, as specified earlier, both returns are defined in terms of the home consumption good. All the equations of the model and the steady state calculation are grouped in the appendix section A.

4 Solution method

It is easy to show why neither the non-stochastic steady state nor a first-order approximation of the model provide enough equations to tie down the zero or first-order components of $\alpha_{E,t}$.

First, in the non-stochastic equilibrium, equations (3)-(4) imply $r_E = r_{E^*}$, i.e. all assets pay the same rate of return. This implies that, for a given wealth W, all portfolio allocations pay the same return, so any value of $\alpha_{E,t}$ is consistent with equilibrium. Thus the non-stochastic steady state does not tie down a unique portfolio allocation.

Second, in a first-order approximation of the model, equations (3)-(4) imply

$\mathbb{E}_t\left[\hat{r}_{E,t+1}\right] = \mathbb{E}_t\left[\hat{r}_{E^*,t+1}\right]$, i.e. all assets have the same expected rate of return. Again, any value of $\alpha_{E,t}$ is consistent with equilibrium.

The problem is easy to state in economic terms. Since in the non-stochastic steady state there is, by definition, no risk, while in a first-order approximation there is certainty equivalence, neither the non-stochastic steady state nor the first-order approximation capture the different risk characteristics of assets. Thus, in both cases, assets cannot be distinguished.

The Devereux and Sutherland (2007) algorithm is based on the work of Samuelson (1970), who established that, in order to derive the zero-order component of the portfolio, it is necessary to approximate the portfolio problem up to the second order. While Samuelson approached the problem by approximating the agent's utility function, Devereux and Sutherland take approximations of agents' first-order conditions.

The log-linearization of the main equilibrium conditions of the model is presented in the appendix section B. Following Devereux and Sutherland (2007), the equation which pins down the solution for optimal portfolios is

$$\mathbb{E}_t\left[\left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}\right)\hat{r}_{x,t+1}\right] = 0 + \mathcal{O}\left(\varepsilon^3\right)$$

The problem is then reduced to find the matrices R_1, R_2, D_1, D_2 which satisfy:

$$\begin{aligned} \hat{r}_{x,t+1} &= R_1\xi_{t+1} + R_2\varepsilon_{t+1} + \mathcal{O}\left(\varepsilon^2\right) \\ \left(\hat{C}_{t+1} - \hat{C}^*_{t+1} - \frac{\hat{Q}_{t+1}}{\rho}\right) &= D_1\xi_{t+1} + D_2\varepsilon_{t+1} + D_3\begin{bmatrix} x_t \\ s_{t+1}\end{bmatrix} + \mathcal{O}\left(\varepsilon^2\right) \end{aligned}$$

Note that we do not need D_3 in what follows (although we can compute it). ξ_{t+1} replaces $\tilde{\alpha}'\hat{r}_{x,t+1}$ and is treated *temporarily* as an exogenous i.i.d. variable. Once matrices R_1, R_2, D_1, D_2 are found, the equilibrim portfolio choice is found in closed form:

$$\tilde{\alpha} = \left[R_2\Sigma D_2' R_1' - D_1 R_2 \Sigma R_2'\right]^{-1} R_2\Sigma D_2' \tag{5}$$

5 Results

The computation of the closed form solution is described in appendix section D. The proportion of home equity held by home residents is

$$\frac{1}{2} + (\rho - 1)\frac{1}{2}\frac{\Psi\left(2\mu - \Psi\right)}{\Gamma} + \frac{\Lambda}{1-\Lambda}\frac{\left(\Psi - \mu\right)\left(2\rho\theta\mu - 2\mu + \Psi - \rho\Psi\right)}{\Gamma} \tag{6}$$

with $\Gamma = \rho\Psi^2 - 4\rho\Psi\mu\theta + 4\rho\mu^2\theta - \Psi^2 + 4\mu\Psi - 4\mu^2$, $\Psi = \mu + (1-\mu)\,t^{1-\theta}$ and Λ represents the share of labor income in total income. This solution is particularly easy to interpret. The first term (1/2) represents the equity portfolio as analyzed by Lucas (1982): in a single-good world with zero labor income, equity portfolios are fully diversified. The second term is identical to the one in Coeurdacier (2006) and represents the hedging of the real exchange rate. This term disappears when there is no consumption home bias ($\mu = 1/2$ and $t = 1$). The sign of this term is analysed by Coeurdacier (2006) and will also be discussed through this section. The third term represents the hedging of the labor income risk. This term will be shown to be negative, which confirms the Baxter and Jermann (1997) result: since labor and capital income are perfectly positively correlated, it will induce foreign equity bias. Naturally, this term will disappear when there is no labor income ($\Lambda = 0$).

In what follows I assume the relative risk aversion ρ to be greater than 1 and the elasticity of subsitution between home and foriegn goods, θ, to be also greater than 1. This is in line with empirical estimates. Let's focus on the second term. First, it can be shown (see appendix section E) that the numerator is a number between 0 and 1. It increases with the preference parameter for home goods μ and with trade costs t. Thus it can be interpreted as a monotonic transformation of barriers to trade in goods. When $\Psi(2\mu - \Psi)$ is 0, then there are no barriers to trade in goods. When $\Psi(2\mu - \Psi)$ is 1, then the markets are segmented. This term is similar to θ found in Coeurdacier (2006). The denominator depends on four parameters: μ, t, ρ, and θ. Figure 1 shows this term for different values of μ and t, when $\rho = \{2, 5\}$ and $\theta = \{1.2, 2, 3\}$.

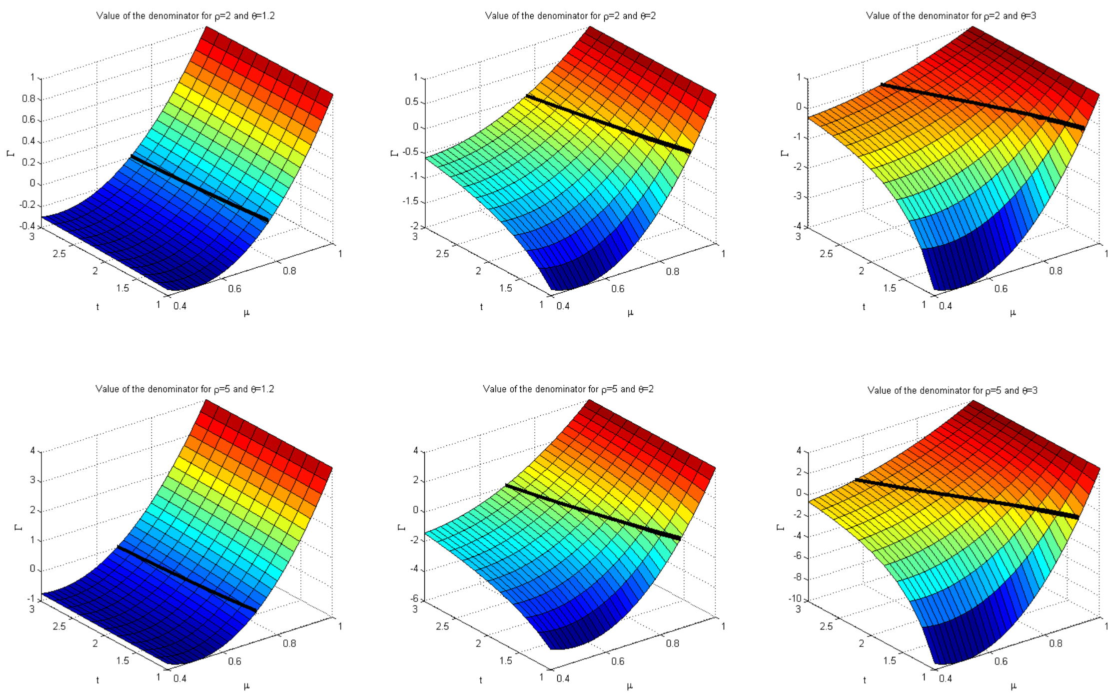

Figure 1: The denominator Γ in (6) for different values of μ, t, ρ and θ. The bold black line in each graph represents the combinations $\{\mu, t\}$ for which $\Gamma = 0$. These lines can be interpreted as a generalization of Cole and Obstfeld (1991).

The bold black line in each graph represents the combinations $\{\mu, t\}$ for which $\Gamma = 0$. In these cases the portfolio is indeterminate. This can be interpreted as an extension of Cole and Obstfeld (1991): home and foreign equities have perfectly correlated returns. In this case, an increase in home output is exactly offset by the response ot the home terms-of-trade. As Coeurdacier (2006) points out, the cases $\Gamma > 0$ are unrealistic for plausible parameter values. This can also be seen in figure 1. Which means that, for most parameter values, the second term will be negative and thus will generate foreign equity bias.

For the third term, it can be shown (see appendix section E) that the numerator is always positive. Same discussion as before applies for the denominator. It then follows that this term is also negative and generate foreign equity bias. This comes in line with Baxter and Jermann (1997).

It results that this setup will not be able to generate home equity bias, due to hedging of the changes in the real exchange rate (second term) and of the changes in the labor income (third term).

This is a quite general setup: it is able to encompass several works done so far in the existing literature. The special cases below will prove this fact and will help us to get more intuition.

5.1 Special case: Lucas (1982)

Let us fix $\mu = 1/2$, no trade costs $t = 1$ (no home bias in consumption, $\Psi(2\mu - \Psi) = 0$) and fix labor income share $\Lambda = 0$. It then follows that both the second and the third term will vanish. The optimal portfolio holding of domestic households (as a proportion of home equity) will be 1/2. This is the well-known result of Lucas (1982), who states that, in equilibrium, all households hold identical equity portfolios, as this permits full risk sharing.

5.2 Special case: Baxter and Jermann (1997)

Let us fix $\mu = 1/2$ and no trade costs $t = 1$ (no home bias in consumption, $\Psi(2\mu - \Psi) = 0$). It follows that the second term dissapears. The optimal portfolio holding as a proportion of home equity becomes

$$\frac{1 - 2\Lambda}{2(1 - \Lambda)},$$

which is exactly the formula (2) in Baxter and Jermann (1997). For example, if labor income is 2/3 of total income, then the optimal choice willl be $-1/2$: households short home equity in order to hedge for labor income risk. This is a well known result due to the fact that, in this model, capital income and labor income are perfectly positively correlated.

5.3 Special case: Cole and Obstfeld (1991)

Let us fix $\mu = 1/2$, and $\theta = 1$ (unit elasticity of substitution across home and foreign goods). It follows that, for any level of the trade costs t, Ψ will be equal to 1. Then we will get indeterminancy of portfolio holdings ($\Gamma = 0$): trade in goods alone will ensure full risk-sharing across countries. This is the Cole and Obstfeld (1991) case.

However, this case can be generalized: as shown before, for every combination of parameters $\{\mu, t, \rho, \theta\}$ which yields $\Gamma = 0$, we will have indeterminancy of portfolio holdings.

5.4 Special case: Kollmann (2006)

Kollmann (2006) builds a similar model, except that there are no trade costs ($t = 1$) and no labor income ($\Lambda = 0$). It follows that $\Psi = 1$ and $\Gamma = \rho - 1 + 4\mu - 4\mu^2 - 4\rho\mu\theta + 4\rho\mu^2\theta$. The optimal portfolio will be

$$\frac{1}{2} + \frac{1}{2}\frac{(\rho - 1)(2\mu - 1)}{\rho - 1 + 4\mu - 4\mu^2 - 4\rho\mu\theta + 4\rho\mu^2\theta} = \frac{\mu(1 - 2\mu + 2\mu\rho\theta + \rho - 2\rho\theta)}{\rho - 1 + 4\mu - 4\mu^2 - 4\rho\mu\theta + 4\rho\mu^2\theta}$$

which is exactly the Kollmann (2006) result (equations (25), (24a) and (24b) in his paper).

5.5 Special case: Coeurdacier (2006)

If we fix labor income to zero ($\Lambda = 0$), then the model replicates the results of Coeurdacier (2006). The optimal portfolio is

$$\frac{1}{2} + (\rho - 1)\frac{1}{2}\frac{\Psi(2\mu - \Psi)}{\Gamma} \tag{7}$$

which is exactly as equation (8) in his paper. This result comes also in line with the findings of Uppal (1993), who shows that in order to obtain home equity bias, investors have to be less risk-averse than log. This is the case here: for reasonable parameter values, the second term in (7) will be positive only if $\rho < 1$ (investors are less risk-averse than log). Proportional trade costs cannot generate home equity bias for investors more risk-averse than log.

5.6 Difficulties to explain the home equity bias

So far I have builded a general model able to replicate some of the well known results in the existing literature. It is obvious at this point that this benchmark model cannot generate home equity bias: both second and third terms in (6) are negative for parameter values in line with empirical estimates.

To gain more intuition why, consider the figure 2. The left panel shows the optimal portfolio in Coeurdacier (2006) case, for different values of trade costs (other parameters are $\rho = 2, \mu = 1/2, \theta = 1.1$). As explained above, the second term in (7) is negative and increasing (in absolute value) with trade costs. Which means that increasing trade costs will lead to foreign equity bias.

Take now the right panel. In this case we insert also labor income. If the share of capital income is one, then we obtain exactly the line in the left-hand side. If the share of capital income is less than one, then the proportion of home equity held by home investors will be changed due to the third term in (6). Basically, this means that $\Lambda > 0$ and thus the third term in (6) will decrease even more the proportion in home equity held by home households. The home equity bias puzzle is worsened, as in Baxter and Jermann (1997).

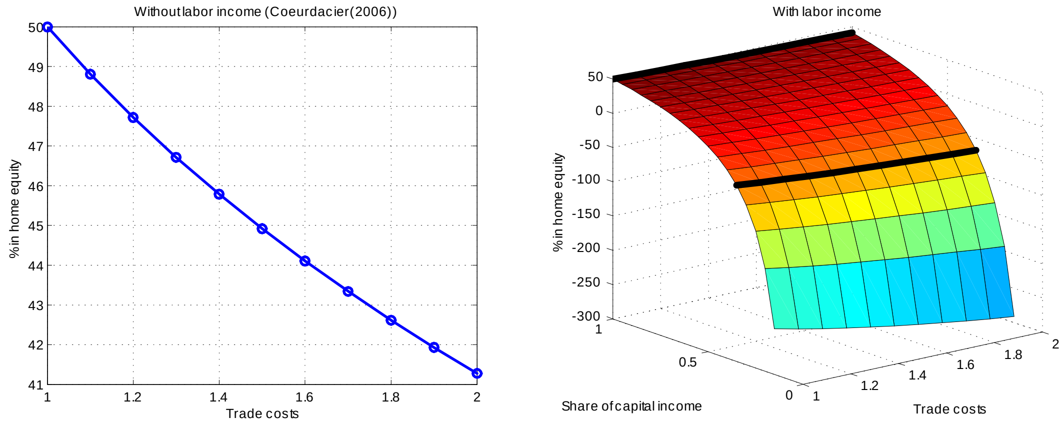

Figure 2: The shortcomings of the benchmark model in explainging the home equity bias. Left panel: The special case of Coeurdacier (2006) for different values of the trade costs. Right panel: Inserting labor income will generate a huge foreign equity bias. Other parameters are $\rho = 2$, $\mu = 1/2$, and $\theta = 1.1$. The two bold black lines in the right panel can be interpreted as follows. The upper line represent the portfolio solution if there is no labor income (same line as in left panel). The second line represents the portfolio solution for a share of 35% labor income. It takes values from -43% if trade costs equals 1 to -68% if trade costs equals 2.

Why is it so difficult for the benchmark model to explain the home equity preference? There are two key points to understand portfolio biases. First, as shown by van Wincoop and Warnock (2006), when the covariance between home excess returns and the home real exchange rate is positive, home investors will prefer home equities as they provide higher returns when the relative price of home goods is higher. It can be shown that the second term in (7) can be writed as

$$\frac{1}{2}\left(1-\frac{1}{\rho}\right)\frac{cov_t\left(\hat{r}_{x,t+1},\hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)},$$

which is similar to the one obtained by van Wincoop and Warnock (2006). They argue that this covariance-variance ratio is very close to zero in the data for the US. Thus, the hedging term due to real exchange rate fluctuations should disappear. In the benchmark model, this term is negative and large in absolute value. Table 1 show the value of this term for different parametrizations of the model (the VWW statistic is defined as $\frac{cov_t\left(\hat{r}_{x,t+1},\hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)}$, see column 3 of the table).

Second, the benchmark model is not able to generate correlation less than 1 between labor income and dividend income. To see why this happens, I show below the results for labor and dividend income respectively:

θ, t	% home equity	VWW	$\rho\left(w_t L_t, N_D \pi_t\left(h\right)\right)$	$\rho\left(\Delta\hat{C}_t, \hat{Q}_t\right)$
$\theta = 1.1, t = 1$	-42.9%	0		
$\theta = 1.1, t = 1.35$	-53.6%	-0.15		
$\theta = 1.1, t = 1.6$	-59.7%	-0.24		
$\theta = 1.3, t = 1$	-42.9%	0		
$\theta = 1.3, t = 1.35$	-53.6%	-0.15	1	-1
$\theta = 1.3, t = 1.6$	-59.8%	-0.24		
$\theta = 1.5, t = 1$	-42.9%	0		
$\theta = 1.5, t = 1.35$	-53.7%	-0.15		
$\theta = 1.5, t = 1.6$	-60.0%	-0.24		

Table 1: Results for the benchmark case. The optimal holdings in home equities for different values of the elasticity of substitution between home and foreign goods and trade costs are shown in column 2. Column 3 shows the VWW statistic $\frac{cov_t\left(\hat{r}_{x,t+1}, \hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)}$ in each case. Columns 4 and 5 show the correlation between labor income and capital income, and the correlation between relative consumption and the real exchange rate. All results are computed for a capital income share of 35 percent.

$$
\begin{aligned}
w_t L_t &= \left(1 - \frac{1}{\phi}\right) L_t A_t \frac{p_t\left(h\right)}{P_t} = \Lambda L_t A_t \frac{p_t\left(h\right)}{P_t} \\
N_D \pi_t\left(h\right) &= \frac{1}{\phi} N_D Y_t\left(h\right) \frac{p_t\left(h\right)}{P_t} = \frac{1}{\phi} L_t A_t \frac{p_t\left(h\right)}{P_t} = \left(1 - \Lambda\right) L_t A_t \frac{p_t\left(h\right)}{P_t}
\end{aligned}
$$

It follows directly from here that the correlation between labor and dividend income is equal to one. Thus, as households seek to hedge their human capital risk, they will sell home equity. The presence of labor income worsens the home bias in equities puzzle.

Another considerable mismatch with the empirical literature is related to the complete markets assumption. Complete markets models counterfactually predict perfect correlation between home to foreign marginal utilities of consumption and the real exchange rate (see Backus and Smith (1993)). As recently documented by Corsetti et al. (2004) and Benigno and Thoenissen (2004), the correlation between relative consumption and the real exchange rate is low in the data.

To conclude, the home equity bias is hard to explain even if this is a quite general setup. In the next section I will consider the extension to a non-tradable sector, fixed export costs and heterogeneous firms. These features will considerably change the results and will bring potential candidates to help us reconcile theory with the home equity bias puzzle.

6 Heterogeneous Firms

6.1 A New Setup

In this section, I add firm heterogeneity in productivity, as well as fixed costs associated with exporting, f_X. These two features will bring selection among exporters: less productive firms cannot generate enough profits abroad to cover the fixed export costs. Thus, only a fraction of the total number of domestic firms will go to export. These sunk costs offer a common justification to the fact that exporters are generally more productive than non-exporters. Melitz (2003) offers a theoretical justification of this selfselection.

The importance of the existence of sunk costs in the export markets is a relatively recent and growing area of interest in the economic literature. Example of sunk costs in exporting could be related to information gathering on the new market, setting up new distribution networks, marketing and possibly repackaging the product to appeal new consumers, adiministrative burdens, etc. Several works have documented the importance of such export market entry costs. Das et al. (2007) estimate an empirical model with marginal and fixed export costs heterogeneity based on panel data for Columbian chemical producers. They conclude that sunk costs vary considerably across plants. Also, surveys reveal that managers making export related decisions are much more concerned with export costs that are fixed in nature rather than high per-unit costs. Furthermore, Jensen and Bernard (2001) estimate that the magnitude of sunk export market entry costs is important enough to generate very large hysteresis effects associated with a plant's export market participation.

In what follows, I will adopt the firm heterogeneity setup from Ghironi and Melitz (2005). It assumes that firm productivity is drawn from a Pareto distribution with shape parameter k and with lower bound $z_{\min}$[6]. A firm's total profit increases with its productivity. Therefore, only firms which have productivity above a cutoff level $z_X > z_{\min}$ will export. Exporters are then a more productive subset of domestic firms. A graph in appendix section C illustrates an example.

I will add these features in two steps. First, I will keep fixed the sunk export costs f_X and the number of exporters, N_X. I will show that a productivity shock will also create a redistributive shock (will affect the redistribution of income between labor and dividends). Thus, the setup can actually have a correlation between labor income and dividend income which is less than 1, without inserting exogenously any other shock, as in Coeurdacier et al. (2007).

Second, I will allow fixed export costs f_X to vary exogenously. We will have then 4 kinds of shocks and 2 assets, and markets will be incomplete. However, the methodology developped by Devereux and Sutherland (2007) still allows me to compute optimal portfolios. I will show that the results are much more robust to changes in parameters and that the incomplete markets setup will give better results related to the "consumption-real exchange rate anomaly" of Backus and Smith (1993).

Finally, I will allow also the number of exporters N_X to vary. This is a natural

[6]The density of the Pareto distribution, as well as the raw moments are presented in the appendix section C.

way to introduce demand shocks as in Coeurdacier et al. (2007), through variety effects. Moreover, this offers a logical setup in terms of the correlations between these three kinds of shocks. For example, if the aggregate productivity is increasing, then more firms will be able to export and thus we should have a positive correlation between productivity shocks and the number of exporters. Also, an increase in the fixed export costs should force some firms to stop their exporting activity, thus a natural negative correlation should be between shocks to exporting costs and shocks to the number of exporting firms.

Due to the fixed export costs, only more productive firms will be able to generate profits by exporting. It then results that there exist a productivity threshold, z_X, for the less productive firm in the home country able to export in the foreign country. This productivity threshold is fixed to match the zero-export profits cutoff condition at steady state.

As in Ghironi and Melitz (2005), the fixed export costs are measured in units of effective labor. Firms need to hire workers from the domestic labor markets to cover these fixed costs. This means that there will be some modifications to the system of equations for the model with heterogeneous firms and export costs. Basically, the labor market clearing will need an extra equation. The new steady state calculation necessites some adaptations. All the technical details are exposed in appendix section F.

6.2 Results for non-time varying f_X and N_X

The appendix section G solves for the closed form solution. I will expose intuitively the differences with the benchmark model. First, the total amount of labor in the economy, L_t, will be divided in two parts: the production part l_t (this was the only part in the benchmark model) and the units of labor paid as fixed costs by the exporting firms, $\frac{N_X f_X}{A_t}$. This division of labor will induce a difference in the dynamics of the terms of trade:

$$\left(1-\frac{\mu}{\Theta}\right)\hat{\tau}_t = \left[1+\frac{1}{\varphi}\left(2\frac{l}{L}-1\right)\right]\hat{A}_t - \rho\hat{C}_t - \frac{1}{\varphi}\hat{Y}_t$$

with $\Theta = \left\{\mu + (1-\mu)\,\chi^{\frac{1-\theta}{1-\phi}}\left(\frac{t}{\eta}\right)^{1-\theta}\right\}$ equivalent to Ψ in the benchmark model, except that it takes into account the proportion of exporting firms χ. Note that the only difference with the benchmark model is the presence of the term $\left(2\frac{l}{L}-1\right)$. If all the amount of labor is used for production (there are no fixed export costs), then $l = L$ and this term becomes 1. In this case, we obtain exactly the benchmark model. Productivity shocks propagate now less strongly to terms of trade movements, since $2\frac{l}{L}-1<1$.

The equation of profits need also some modifications. At the steady state, the total profits in the home country are (see proof in appendix section G):

$$\pi = \frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}(h)}{P} - N_X\frac{wf_X}{A}$$

Intuitively, the first term on the right hand side is the total profit if there are no fixed costs for exporting. This corresponds to the benchmark case. Then, these fixed costs are substracted to obtain the net total profits in the home economy. After some manipulations, wich are shown in appendix section G, we obtain the following dynamics for the total profits in the home economy.

$$\hat{\pi}_t = \frac{\frac{1}{\phi} N_D \tilde{Y} \frac{\tilde{p}(h)}{P}}{\pi} \hat{Y}_t + \left(\frac{\mu}{\Theta} - 1\right) \hat{\tau}_t = \Upsilon \hat{Y}_t + \left(\frac{\mu}{\Theta} - 1\right) \hat{\tau}_t,$$

with $\Upsilon = \frac{\frac{1}{\phi} N_D \tilde{Y} \frac{\tilde{p}(h)}{P}}{\pi}$. This equation is similar to its correspondent in the benchmark case, except one term Υ. We recognize easily that this term is larger or equal to 1. In the case when the term is equal to 1 (no fixed export costs), then we will have exactly the same equation as in the benchmark case.

At this point, we can have a new look on the correlation between labor income and dividend income. Compared to the benchmark case, they are defined now as follows.

$$\begin{aligned} w_t L_t &= w_t \left(l_t + \frac{N_X f_X}{A_t}\right) = \left(1 - \frac{1}{\phi}\right) \tilde{z}_D A_t l_t \frac{\tilde{p}_t(h)}{P_t} + w_t \frac{N_X f_X}{A_t} \\ N_D \pi_t(h) &= \frac{1}{\phi} \tilde{z}_D A_t l_t \frac{\tilde{p}_t(h)}{P_t} - w_t \frac{N_X f_X}{A_t} \end{aligned}$$

The first terms in both equations correspond to the ones in the benchmark case. Then, we have new terms due to fixed export costs ($w_t \frac{N_X f_X}{A_t}$). The presence of these terms show that a shock in productivity will make these two incomes to move in opposite directions. Thus, this will decrease the correlation between labor and dividend income.

All the computations needed to obtain the closed form solution are exposed in appendix section G. The optimal proportion of home equity held by home residents is

$$\frac{1}{2} + \frac{(\rho - 1)}{2} \frac{\Theta (2\mu - \Theta)}{\Sigma} + (\phi - 1) \frac{(\Theta - \mu)(2\theta\mu\rho - 2\mu + \Theta - \rho\Theta)}{\Sigma} + \frac{\rho \left(\frac{1}{\Upsilon} - 1\right)}{2} \frac{\Theta^2}{\Sigma} \quad (8)$$

with Θ defined before, and Σ similar to Γ found in the benchmark economy:

$$\Sigma = 4\mu^2 \theta\rho - 4\mu^2 - 4\mu\rho\Theta\theta + 4\mu\Theta + \frac{\rho}{\Upsilon}\Theta^2 - \Theta^2$$

Note that in this case the share of capital income in total income is defined as

$$\frac{\pi}{N_D \tilde{Y} \frac{\tilde{p}(h)}{P}} = \frac{1}{\Upsilon\phi}$$

It then follows that the share of labor income Λ will be equal to $1 - \frac{1}{\Upsilon\phi}$. The closed form solution is very similar to the one obtained in the benchmark case, except that Υ is entering in the definition of Σ. We notice also the presence of a new term which will modify the hedging of the labor income risk. If we come back to the benchmark case, then Υ is equal to 1 and this fourth term dissapears. We recognize then in this formula the same components as before. First, the Lucas (1982) term, then the term related to the hedging of the real exchange rate. The last two terms are related to the hedging of the labor income risk.

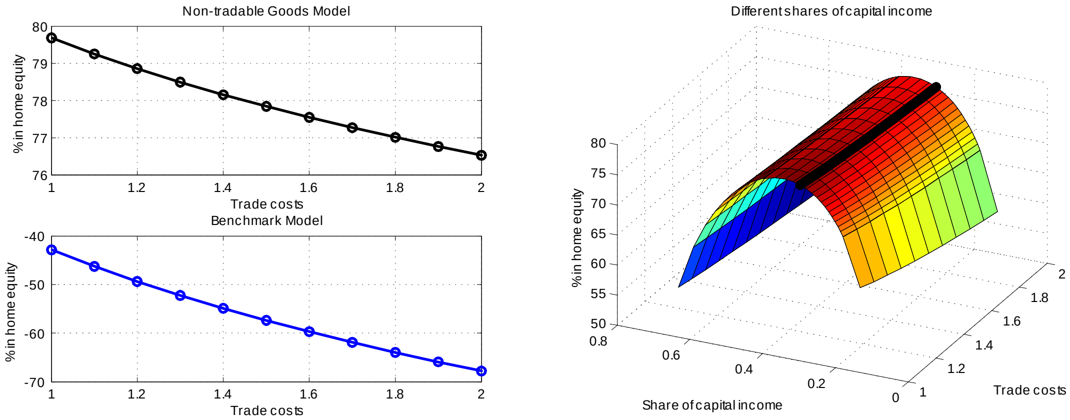

Figure 3: Results of the model with fixed export costs and a non-tradable sector. The share of capital income is calibrated to 35% in the left panels and is varying in the right panel. The fixed export costs f_X are calibrated to match a share of 20% exporting firms. The parameter k of the Pareto distribution is calibrated to match the standard deviation of log U.S. plant sales, 1.67, reported by Bernard et al. (2003). Other parameters are $\rho = 2$, $\mu = 1/2$, and $\theta = 1.1$. The bold black line in the right graph represents the solution for 35% capital income share.

All the parameters are calibrated as in the benchmark model. Then, f_X is calibrated to match a share of 20% exporting firms. The parameter k of the Pareto distribution is calibrated to match the standard deviation of log U.S. plant sales, 1.67, reported by Bernard et al. (2003). The share of capital income in total income is fixed to 35%. Other parameters are $\rho = 2$, $\mu = 1/2$, $\theta = 1.1$. The results are exposed in the Figure 3. The two left panels show the solution of the non-tradable goods model for different levels of trade costs. The other panel shows the solution given by the benchmark model. The interaction between fixed export costs, heterogeneous firms and the non-tradable sector is successful in explaining the home equity bias. The right panel shows the results if several shares of capital income are considered. The bold black lines correspond to the one in the left panel, for a share of 35% capital income.

A similar analysis in terms of VWW statistic, labor/dividend income correlation and relative consumption/RER correlation is conducted in table 2. The strong correlation between labor and dividend income has been decreased, but only for small values of the elasticity of substitution θ. And we still face the problem of the perfect negative correlation between relative consumption and the real exchange rate, since markets remain complete.

The intuition for these results is as follows. An increase in the home aggregate productivity will decrease the amount of labor needed for the sunk export costs. This will lead to a decrease in the domestic labor income. In the same time, the same shock will lead to a increase in the dividends of domestic firms. The productivity

θ, t	% home equity	VWW	$\rho\left(w_t L_t, N_D \pi_t\left(h\right)\right)$	$\rho\left(\Delta\hat{C}_t, \hat{Q}_t\right)$
$\theta = 1.1, t = 1$	79.7%	-0.06	0.37	
$\theta = 1.1, t = 1.35$	78.3%	-0.08	0.40	
$\theta = 1.1, t = 1.6$	77.6%	-0.09	0.42	
$\theta = 1.3, t = 1$	46.7%	-0.10	0.82	
$\theta = 1.3, t = 1.35$	42.2%	-0.16	0.86	-1
$\theta = 1.3, t = 1.6$	39.6%	-0.19	0.88	
$\theta = 1.5, t = 1$	23.4%	-0.12	0.94	
$\theta = 1.5, t = 1.35$	16.4%	-0.21	0.96	
$\theta = 1.5, t = 1.6$	12.3%	-0.26	0.97	

Table 2: Results for the fixed export costs case. The optimal holdings in home equities for different values of the elasticity of substitution between home and foreign goods and trade costs are shown in column 2. Column 3 shows the VWW statistic $\frac{cov_t\left(\hat{r}_{x,t+1}, \hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)}$ in each case. Columns 4 and 5 show the correlation between labor income and capital income, and the correlation between relative consumption and the real exchange rate. All results are computed for a capital income share of 35 percent. The model is now able to decrease the correlation between labor and dividend income.

shock will then propagate into the economy both as a supply and as a redistributive shock. Changes in productivity do not affect only the output but also the income distribution in the economy. Therefore, this setup is able to introduce both supply and redistributive shocks in a more structural way than in Coeurdacier et al. (2007).

6.3 Incomplete Markets

The previous setup is still not able to decrease the perfect negative correlation between relative consumption and the real exchange rate. This is due to the perfect risk-sharing condition in complete markets. In this section I will try to deal with this issue by introducing exogenous shocks to the fixed export costs f_X.

Let us come back again to the definition of the labor and capital income. Since $f_{X,t}$ is now time varying, it results from (9)-(10) that this should intuitively lead to further decrease in the correlation labor/capital income.

$$w_t L_t = \left(1 - \frac{1}{\phi}\right) \tilde{z}_D A_t l_t \frac{\tilde{p}_t\left(h\right)}{P_t} + w_t \frac{N_X f_{X,t}}{A_t} \tag{9}$$

$$N_D \pi_t\left(h\right) = \frac{1}{\phi} \tilde{z}_D A_t l_t \frac{\tilde{p}_t\left(h\right)}{P_t} - w_t \frac{N_X f_{X,t}}{A_t} \tag{10}$$

The new results are exposed in table 3. As expected, the incomplete markets setup will eliminate the perfect negative correlation between relative consumption and the real exchange rate. However, this correlation still remains sizable in absolute value compared to the data. Even if it is going in the right direction, the model cannot quantitatively reproduce the low consumption-real exchange rate correlation observed in the data.

The correlation between labor/capital income is less important than the complete

θ, t	% home equity	VWW	$\rho\left(w_t L_t, N_D \pi_t\left(h\right)\right)$	$\rho\left(\Delta\hat{C}_t, \hat{Q}_t\right)$
$\theta = 1.1, t = 1$	86.4%	-0.04	-0.13	-0.72
$\theta = 1.1, t = 1.35$	85.5%	-0.05	-0.10	-0.75
$\theta = 1.1, t = 1.6$	85.0%	-0.06	-0.09	-0.76
$\theta = 1.3, t = 1$	61.1%	-0.08	0.42	-0.63
$\theta = 1.3, t = 1.35$	57.7%	-0.13	0.48	-0.68
$\theta = 1.3, t = 1.6$	55.7%	-0.15	0.52	-0.70
$\theta = 1.5, t = 1$	38.6%	-0.10	0.70	-0.56
$\theta = 1.5, t = 1.35$	32.1%	-0.17	0.76	-0.63
$\theta = 1.5, t = 1.6$	28.3%	-0.22	0.79	-0.67

Table 3: Results for the fixed export costs case, with incomplete markets and time-varying fixed export costs. The optimal holdings in home equities for different values of the elasticity of substitution between home and foreign goods and trade costs are shown in column 2. Column 3 shows the VWW statistic $\frac{cov_t\left(\hat{r}_{x,t+1}, \hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)}$ in each case. Columns 4 and 5 show the correlation between labor income and capital income, and the correlation between relative consumption and the real exchange rate. All results are computed for a capital income share of 35 percent. The model is now able to eliminate the perfect negative correlation between relative consumption and the real exchange rate, due to the incomplete markets setup.

markets setup, and is increasing less significantly with the elasticithy of substitution θ. The model is able to generate home equity preference for a bigger range of parameter values.

Coeurdacier et al. (2007) introduce also demand shocks in their setup. This can be done also here by making the number of exporters N_X time-varying, and creating a demand shock through variety effects. It comes in line with recent empirical evidence at a very desagregated level by Broda and Weinstein (2007) who suggest that varieties changes are an important phenomenon. Moreover, in terms of the correlation between shocks, the setup offers a simple intuition. When productivity increases, more firms will be able to export, thus we should have a positive correlation between productivity shocks and N_X shocks. When fixed export costs f_X are increasing, then less firms should be able to export, therefore we should naturally observe a negative correlation between f_X shocks and N_X shocks.

As an exercise, I fix these correlations to 0.6 and -0.6 respectively. The new results are exposed in table 4. Home equity bias is still persistent for a large range of parameters. In the same time, the model is able to keep low values for the correlation labor/capital income, and to generate low correlations between relative consumption and the real exchange rate.

These results are similar to the ones obtained by Coeurdacier et al. (2007). However, all their shocks are exogenous and in their setup it is difficult to correlate them intuitively. In my case, I showed how a productivity shock can be interpreted botha as a supply and a redistributive shock. My aim is to work more structurally and to provide a setup where shocks are more intuitive. In this way, I am able to relate demand shocks to productivity shocks by introducing varieties and heterogeneous firms.

This setup highlights a simple mechanism which generates a correlation less than

θ, t	% home equity	VWW	$\rho\left(w_t L_t, N_D \pi_t\left(h\right)\right)$	$\rho\left(\Delta \hat{C}_t, \hat{Q}_t\right)$
$\theta = 1.1, t = 1$	88.8%	0.04	0.29	-0.30
$\theta = 1.1, t = 1.35$	87.7%	0.03	0.31	-0.29
$\theta = 1.1, t = 1.6$	87.1%	0.02	0.32	-0.28
$\theta = 1.3, t = 1$	55.6%	-0.08	0.46	-0.08
$\theta = 1.3, t = 1.35$	50.6%	-0.12	0.51	-0.08
$\theta = 1.3, t = 1.6$	47.6%	-0.14	0.54	-0.08
$\theta = 1.5, t = 1$	24.3%	-0.08	0.61	0.01
$\theta = 1.5, t = 1.35$	13.2%	-0.15	0.68	-0.01
$\theta = 1.5, t = 1.6$	6.5%	-0.20	0.72	-0.03

Table 4: Results for the fixed export costs case, with incomplete markets and time-varying fixed export costs and number of exporting firms. The optimal holdings in home equities for different values of the elasticity of substitution between home and foreign goods and trade costs are shown in column 2. Column 3 shows the VWW statistic $\frac{cov_t\left(\hat{r}_{x,t+1}, \hat{Q}_{t+1}\right)}{var_t\left(\hat{r}_{x,t+1}\right)}$ in each case. Columns 4 and 5 show the correlation between labor income and capital income, and the correlation between relative consumption and the real exchange rate. All results are computed for a capital income share of 35 percent.

one between labor income and capital income. The fixed export costs f_X are able to do this, without inserting exogenously redistributive shocks as in Coeurdacier et al. (2007).

7 Conclusion

This paper builds a dynamic stochastic general equilibrium model of international portfolio choice, with home bias in consumption generated by both a preference parameter and trade costs in goods, à la Coeurdacier (2006). Optimal portfolios are computed in closed form, with the method developped by Devereux and Sutherland (2007). It is shown why this benchmark model is unable to explain home equity bias.

Then deviations from the benchmark model are considered by inserting a setup with heterogeneous firms, a non-tradable sector and fixed export costs. These new features are able to explain the home equity bias for reasonable parameter values. The model is then extended to incomplete markets and results are shown to be more robust to parameter changes. The incomplete markets setup helps in explaining the consumption-real exchange rate anomaly (Backus and Smith (1993)).

In comparison with similar studies, such as Coeurdacier et al. (2007), the paper explain how the fixed export costs generate a simple mechanism which brings a correlation less than one between labor income and capital income. Demand shocks are intoduced naturally into the model, through variety effects. Finally, the setup offers a clear understanding of the correlations between the shocks in the economy.

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Appendix A

The benchmark model

$$W_t = \alpha_{E,t-1} r_{E,t} + \alpha_{E^*,t-1} r^*_{E,t} + N_D Y_t(h) \frac{p_t(h)}{P_t} - C_t \tag{11}$$

$$N_D^{\frac{\theta-\phi}{1-\phi}} Y_{D,t}(h) \left(\frac{p_t(h)}{P_t}\right)^{\theta} = \mu C_t \tag{12}$$

$$N_D^{\frac{\theta-\phi}{1-\phi}} Y_{X,t}(h) \left(\frac{p^*_t(h)}{P^*_t}\right)^{\theta} \frac{1}{t} = (1-\mu) C^*_t \tag{13}$$

$$Y_t(h) = Y_{D,t}(h) + Y_{X,t}(h) \tag{14}$$

$$\frac{p_t(h)}{P_t} = \frac{\phi}{\phi-1} \frac{w_t}{A_t} \tag{15}$$

$$N_D Y_t(h) = A_t L_t \tag{16}$$

$$w_t = \kappa C_t^{\rho} L_t^{\frac{1}{\varphi}} \tag{17}$$

$$\pi_t(h) = \frac{1}{\phi} Y_t(h) \frac{p_t(h)}{P_t} \tag{18}$$

$$C_t^{-\rho} = \beta \mathbb{E}_t \left[C_{t+1}^{-\rho} r_{E,t+1}\right] \tag{19}$$

$$C_t^{-\rho} = \beta \mathbb{E}_t \left[C_{t+1}^{-\rho} r^*_{E,t+1}\right] \tag{20}$$

$$r_{E,t} = \frac{\pi_t(h) + v_t}{v_{t-1}} \tag{21}$$

Add equations (12) - (21) for the foreign country:

$$(N^*_D)^{\frac{\theta-\phi}{1-\phi}} Y^*_{D,t}(f) \left(\frac{p^*_t(f)}{P^*_t}\right)^{\theta} = \mu C^*_t \tag{22}$$

$$(N^*_D)^{\frac{\theta-\phi}{1-\phi}} Y^*_{X,t}(f) \left(\frac{p_t(f)}{P_t}\right)^{\theta} \frac{1}{t} = (1-\mu) C_t \tag{23}$$

$$Y^*_t(f) = Y^*_{D,t}(f) + Y^*_{X,t}(f) \tag{24}$$

$$\frac{p^*_t(f)}{P^*_t} = \frac{\phi}{\phi-1} \frac{w^*_t}{A^*_t} \tag{25}$$

$$N^*_D Y^*_t(f) = A^*_t L^*_t \tag{26}$$

$$w^*_t = \kappa (C^*_t)^{\rho} (L^*_t)^{\frac{1}{\varphi}} \tag{27}$$

$$\pi^*_t(f) = \frac{1}{\phi} Y^*_t(f) \frac{p^*_t(f)}{P^*_t} \tag{28}$$

$$(C^*_t)^{-\rho} = \beta \mathbb{E}_t \left[\frac{Q_t}{Q_{t+1}} \left(C^*_{t+1}\right)^{-\rho} r_{E,t+1}\right] \tag{29}$$

$$(C^*_t)^{-\rho} = \beta \mathbb{E}_t \left[\frac{Q_t}{Q_{t+1}} \left(C^*_{t+1}\right)^{-\rho} r^*_{E,t+1}\right] \tag{30}$$

$$r^*_{E,t} = \frac{\pi^*_t(f) + v^*_t}{v^*_{t-1}} \frac{Q_{t-1}}{Q_t} \tag{31}$$

Steady-state calculation

Variables without time subscript represent steady-state values. The aggregate consumer price index is

$$
\begin{aligned}
P_t^{1-\theta} &= \mu P_{H,t}^{1-\theta} + (1-\mu) P_{F,t}^{1-\theta} \\
&= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu \left[p_t(h) \right]^{1-\theta} + (1-\mu) \left[p_t(f) \right]^{1-\theta} \right\}
\end{aligned}
$$

Divide by $[p_t(h)]^{1-\theta}$

$$
\begin{aligned}
\left[\frac{P_t}{p_t(h)} \right]^{1-\theta} &= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) \left[\frac{p_t(f)}{p_t(h)} \right]^{1-\theta} \right\} \\
&= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) t^{1-\theta} \left[\frac{p_t^*(f)}{p_t(h)} \right]^{1-\theta} \right\}
\end{aligned}
$$

In a symmetrical steady state we have $\frac{p^*(f)}{p(h)} = 1$, thus we obtain

$$
\begin{aligned}
\left[\frac{P}{p(h)} \right]^{1-\theta} &= N_D^{\frac{1-\theta}{1-\phi}} \left[\mu + (1-\mu) t^{1-\theta} \right] \\
&= N_D^{\frac{1-\theta}{1-\phi}} \Psi
\end{aligned}
$$

with $\Psi = \left\{ \mu + (1-\mu) t^{1-\theta} \right\}$. We obtain therefore

$$
\frac{p(h)}{P} = N_D^{\frac{1}{\phi-1}} \Psi^{\frac{1}{\theta-1}} \tag{32}
$$

We fix $Y(h) = 1$ and $A = 1$. From (11), (12) and (13) we get

$$
Y_D(h) = \frac{\mu}{\Psi} Y(h), \quad Y_D(h) = \left(1 - \frac{\mu}{\Psi} \right) Y(h) = \frac{1-\mu}{\Psi t^{\theta-1}} Y(h)
$$

We find that (14) is automatically verified. The steady state calculation reduces to the following system of equations:

$$
\begin{cases}
N_D Y(h) \frac{p(h)}{P} = C & \text{from (11)} \\
\frac{p(h)}{P} = \frac{\phi}{\phi-1} \frac{w}{A} & \text{from (15)} \\
N_D Y(h) = AL & \text{from (16)} \\
w = \kappa C^\rho L^{\frac{1}{\varphi}} & \text{from (17)}
\end{cases}
$$

Replace (32) to obtain a system with 4 equations and 4 unknowns (C, N_D, w, L):

$$
\begin{cases}
N_D^{\frac{\phi}{\phi-1}} \Psi^{\frac{1}{\theta-1}} = C \\
N_D^{\frac{1}{\phi-1}} \Psi^{\frac{1}{\theta-1}} = \frac{\phi}{\phi-1} w \\
N_D = L \\
w = \kappa C^\rho L^{\frac{1}{\varphi}}
\end{cases}
$$

Solve first for N_D

$$
N_D = \left[\kappa \frac{\phi}{\phi-1} \Psi^{\frac{\rho-1}{\theta-1}} \right]^{\frac{1}{\frac{1-\rho\phi}{\phi-1} - \frac{1}{\varphi}}}
$$

Then the other values follow. From (18)-(21), we find $r_E = r_E^* = 1/\beta$, $\pi(h) = \frac{1}{\phi} Y(h) N_D^{\frac{1}{\phi-1}} \Psi^{\frac{1}{\theta-1}}$, and $v = \frac{\beta}{1-\beta} \pi(h)$.

Appendix B

Log-linearization

We need to define first $\hat{p}_t(h) - \hat{P}_t, \hat{p}^*_t(h) - \hat{P}^*_t$, and $\hat{Q}_t$ in terms of variations of the terms of trade, which I define as $\hat{\tau}_t = \hat{p}^*_t(f) - \hat{p}_t(h)$. From appendix section A, we know that

$$\left[\frac{P_t}{p_t(h)}\right]^{1-\theta} = N_D^{\frac{1-\theta}{1-\phi}}\left\{\mu + (1-\mu)\, t^{1-\theta}\left[\frac{p^*_t(f)}{p_t(h)}\right]^{1-\theta}\right\}$$

After some manipulations we obtain

$$\begin{aligned}
\hat{p}_t(h) - \hat{P}_t &= \left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t \\
\hat{p}_t(f) - \hat{P}_t &= \frac{\mu}{\Psi}\hat{\tau}_t \\
\hat{p}^*_t(f) - \hat{P}^*_t &= -\left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t \\
\hat{p}^*_t(h) - \hat{P}^*_t &= -\frac{\mu}{\Psi}\hat{\tau}_t \\
\hat{Q}_t &= \left(\frac{2\mu}{\Psi} - 1\right)\hat{\tau}_t
\end{aligned}$$

As in Devereux and Sutherland (2007), I define $\hat{W}_t = \frac{W_t - W}{N_D Y(h)\frac{p(h)}{P}}$, and $\tilde{\alpha} = \frac{\bar{\alpha}}{\beta N_D Y(h)\frac{p(h)}{P}}$. With this notation, the log-linearization of the model equations becomes

$$\hat{W}_t = \frac{1}{\beta}\hat{W}_{t-1} + \hat{Y}_t(h) + \left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t - \hat{C}_t + \tilde{\alpha}\hat{r}_{x,t} \tag{33}$$

$$\rho\hat{C}_t - \rho\mathbb{E}_t\left[\hat{C}_{t+1}\right] = \rho\hat{C}^*_t - \rho\mathbb{E}_t\left[\hat{C}^*_{t+1}\right] + \left(\frac{2\mu}{\Psi} - 1\right)\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) \tag{34}$$

$$\hat{Y}_{D,t}(h) + \theta\left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t = \hat{C}_t \tag{35}$$

$$\hat{Y}_{X,t}(h) - \theta\frac{\mu}{\Psi}\hat{\tau}_t = \hat{C}^*_t \tag{36}$$

$$\hat{Y}_t(h) = \frac{\mu}{\Psi}\hat{Y}_{D,t}(h) + \left(1 - \frac{\mu}{\Psi}\right)\hat{Y}_{X,t}(h) \tag{37}$$

$$\left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t = \hat{w}_t - \hat{A}_t \tag{38}$$

$$\hat{Y}_t(h) = \hat{A}_t + \hat{L}_t \tag{39}$$

$$\hat{w}_t = \rho\hat{C}_t + \frac{1}{\varphi}\hat{L}_t \tag{40}$$

$$\hat{\pi}_t(h) = \hat{Y}_t(h) + \left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t \tag{41}$$

$$(-\rho)\,\hat{C}_t = -\rho\mathbb{E}_t\left[\hat{C}_{t+1}\right] + \mathbb{E}_t\left[\hat{r}_{E,t+1}\right] \tag{42}$$

$$\hat{r}_{E,t} = (1-\beta)\,\hat{\pi}_t(h) + \beta\hat{v}_t - \hat{v}_{t-1} \tag{43}$$

Add the equations corresponding to the foreign country:

$$\hat{Y}^{*}_{D,t}(f) - \theta\left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t = \hat{C}^{*}_t \tag{44}$$

$$\hat{Y}^{*}_{X,t}(f) + \theta\frac{\mu}{\Psi}\hat{\tau}_t = \hat{C}_t \tag{45}$$

$$\hat{Y}^{*}_{t}(f) = \frac{\mu}{\Psi}\hat{Y}^{*}_{D,t}(f) + \left(1 - \frac{\mu}{\Psi}\right)\hat{Y}^{*}_{X,t}(f) \tag{46}$$

$$\left(1 - \frac{\mu}{\Psi}\right)\hat{\tau}_t = \hat{w}^{*}_t - \hat{A}^{*}_t \tag{47}$$

$$\hat{Y}^{*}_{t}(f) = \hat{A}^{*}_t + \hat{L}^{*}_t \tag{48}$$

$$\hat{w}^{*}_t = \rho\hat{C}^{*}_t + \frac{1}{\varphi}\hat{L}^{*}_t \tag{49}$$

$$\hat{\pi}^{*}_t(f) = \hat{Y}^{*}_t(f) - \left(\frac{\mu}{\Psi} - 1\right)\hat{\tau}_t \tag{50}$$

$$(-\rho)\hat{C}^{*}_t = -\rho\mathbb{E}_t\left[\hat{C}^{*}_{t+1}\right] + \mathbb{E}_t\left[\hat{r}^{*}_{E,t+1}\right] + \left(\frac{2\mu}{\Psi} - 1\right)\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) \tag{51}$$

$$\hat{r}^{*}_{E,t} = (1-\beta)\hat{\pi}^{*}_t(f) + \beta\hat{v}^{*}_t - \hat{v}^{*}_{t-1} + \left(\frac{2\mu}{\Psi} - 1\right)\left(\hat{\tau}_t - \hat{\tau}_{t-1}\right) \tag{52}$$

Following propositions 1, 2 and 3 from Devereux and Sutherland (2007), I will consider $\hat{\xi}_t = \tilde{\alpha}\hat{r}_{x,t}$ as an exogenous *iid* variable. It then results a system of 20 equations in 20 endogenous variables ($\hat{W}_t$, $\hat{\tau}_t$, $\hat{Y}_t(h)$, $\hat{Y}^{*}_t(f)$, $\hat{C}_t$, $\hat{C}^{*}_t$, $\hat{Y}_{D,t}(h)$, $\hat{Y}^{*}_{D,t}(f)$, $\hat{Y}_{X,t}(h)$, $\hat{Y}^{*}_{X,t}(f)$, $\hat{w}_t$, $\hat{w}^{*}_t$, $\hat{L}_t$, $\hat{L}^{*}_t$, $\hat{\pi}_t$, $\hat{\pi}^{*}_t$, $\hat{r}_{E,t}$, $\hat{r}^{*}_{E,t}$, $\hat{z}_{E,t}$, $\hat{z}^{*}_{E,t}$). There is one endogenous variable predetermined as of time t: the home financial wealth $\hat{W}_t$. Finally, the model features 3 exogenous variables: the aggregate productivities A_t and A^{*}_t and the *iid* shock $\hat{\xi}_t$.

[DETALII DOAR PENTRU TINE]
codul Matlab (simplu) se gaseste in \MODEL_MATLAB\BENCHMARK
a) cu el poti sa faci toate cazurile speciale
b) poti sa faci si impulse responses
codul Matlab (iteratii) se gaseste in \MODEL_MATLAB\BENCHMARK_IT

Appendix C

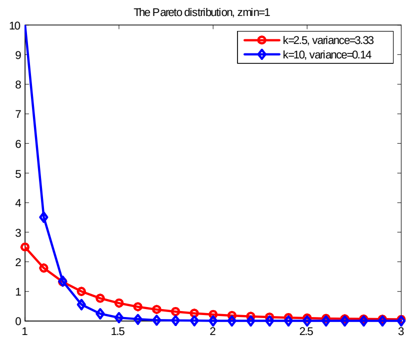
The Pareto Distribution

The density of the Pareto distribution is

$$f(x, k, x_m) = k\frac{x_m^k}{x^{k+1}}, \text{ for } x \geq x_m$$

The expected value of a random variable following a Pareto distribution is $\frac{kx_m}{k-1}$. The variance is $\left(\frac{x_m}{k-1}\right)^2 \frac{k}{k-2}$ and the raw moments are defined as $\mu_n = \frac{kx_m^n}{k-n}$.

Dispersion decreases as k increases and the firm productivity levels are increasingly concentrated toward their lower bound $z_{\min}$. An example is presented in the following figure.

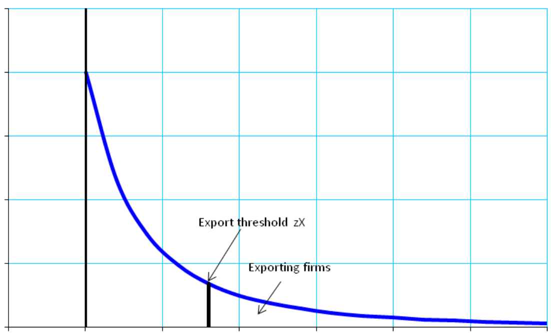


Following Melitz (2003), the average productivity level of all domestic firms is defined as $\tilde{z}_D = z_{\min}\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1}{\phi-1}}$. Average productivity of exporting firms, $\tilde{z}_X$, is equal to $z_X\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1}{\phi-1}}$, where z_X is the zero export profit cutoff level.

I define $\eta = \frac{\tilde{z}_X}{\tilde{z}_D}$. Since exporting firms are more productive in average than all domestic firms, we have $\eta \geq 1$. The proportion of exporting firms is defined as $\chi = \frac{N_X}{N_D}$. Using the definition of the cumulative Pareto distribution, we can find the relationship between η and χ:

$$\begin{aligned}
\chi &= \frac{N_X}{N_D} = 1 - G(z_X) = \left(\frac{z_{\min}}{z_X}\right)^k \\
&= \left(\frac{z_{\min}}{\tilde{z}_X}\right)^k \left[\frac{k}{k-(\phi-1)}\right]^{\frac{k}{\phi-1}} = \left(\frac{\tilde{z}_D}{\tilde{z}_X}\right)^k
\end{aligned}$$

We obtain therefore that $\chi = \eta^{-k}$. The figure below illustrates an example:



Appendix D

Benchmark Model - Closed Form solution

The calculations for the closed form solution are tedious. Everything becomes a lot easier when using some symbolic computation toolbox. In this case, I use the Symbolic Math toolbox of Matlab. Following Devereux and Sutherland (2007), the equation which pins down the solution for optimal portfolios is

$$\mathbb{E}_t\left[\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}\right)\hat{r}_{x,t+1}\right]=0+\mathcal{O}\left(\varepsilon^3\right)$$

It follows that we need to compute $\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}$ and $\hat{r}_{x,t+1}$.

First Term $\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}\right)$

Start with the budget constraints [28] for the home and foreign agents:

$$\begin{aligned}\hat{W}_t &= \frac{1}{\beta}\hat{W}_{t-1}+\hat{Y}_t\left(h\right)+\left(\frac{\mu}{\Psi}-1\right)\hat{\tau}_t-\hat{C}_t+\tilde{\alpha}\hat{r}_{x,t}\\ -\hat{W}_t &= -\frac{1}{\beta}\hat{W}_{t-1}+\hat{Y}^*_t\left(f\right)-\left(\frac{\mu}{\Psi}-1\right)\hat{\tau}_t-\hat{C}^*_t-\tilde{\alpha}\hat{r}_{x,t}\end{aligned}$$

Take the difference of the two budget constraints and write everything at time $t+1$:

$$\hat{W}_t=\beta\hat{W}_{t+1}-\frac{\beta}{2}\mathbb{Y}_{t+1}+\frac{\beta}{2}\mathbb{C}_{t+1}+\beta\left(1-\frac{\mu}{\Psi}\right)\hat{\tau}_{t+1}-\beta\tilde{\alpha}\hat{r}_{x,t+1}, \tag{53}$$

with $\mathbb{Y}_{t+1}=\hat{Y}_{t+1}\left(h\right)-\hat{Y}^*_{t+1}\left(f\right)$ and $\mathbb{C}_{t+1}=\hat{C}_{t+1}-\hat{C}^*_{t+1}$. Use [30 - 35] and [39 - 44] to compute $\mathbb{Y}_{t+1}$. Start with

$$\begin{aligned}\mathbb{Y}_{t+1} &= \hat{Y}_{t+1}\left(h\right)-\hat{Y}^*_{t+1}\left(f\right)\\ &= \left[\frac{\mu}{\Psi}\hat{Y}_{D,t+1}\left(h\right)+\left(1-\frac{\mu}{\Psi}\right)\hat{Y}_{X,t+1}\left(h\right)\right]-\left[\frac{\mu}{\Psi}\hat{Y}^*_{D,t+1}\left(f\right)+\left(1-\frac{\mu}{\Psi}\right)\hat{Y}^*_{X,t+1}\left(f\right)\right]\end{aligned}$$

Then replace $\hat{Y}_{D,t+1}\left(h\right)$, $\hat{Y}_{X,t+1}\left(h\right)$, etc. Replace also from equations [33 - 35] and [43 - 44] $\left(1-\frac{\mu}{\Psi}\right)\hat{\tau}_{t+1}$ as a function of $\mathbb{C}_{t+1}$, $\mathbb{A}_{t+1}$ and $\mathbb{Y}_{t+1}$. After some manipulations we obtain

$$\begin{aligned}\mathbb{Y}_{t+1} &= \left(\frac{2\mu}{\Psi}-1\right)\mathbb{C}_{t+1}+\frac{4\mu\theta}{\Psi}\left(1-\frac{\mu}{\Psi}\right)\hat{\tau}_{t+1}\\ &= \left(\frac{2\mu}{\Psi}-1\right)\mathbb{C}_{t+1}+\frac{2\mu\theta}{\Psi}\left[\left(1+\frac{1}{\varphi}\right)\mathbb{A}_{t+1}-\rho\mathbb{C}_{t+1}-\frac{1}{\varphi}\mathbb{Y}_{t+1}\right],\end{aligned}$$

with $\mathbb{A}_{t+1}=A_{t+1}-A^*_{t+1}$. It then follows that

$$\mathbb{Y}_{t+1}=\left(1/2\,\frac{\Psi\left(-\rho\varphi\Psi-2\mu+\Psi\right)}{\left(\Psi-\mu\right)\left(\Psi\varphi+2\mu\theta\right)}\right)\mathbb{C}_{t+1}+\left(1/2\frac{\Psi^2\left(\varphi+1\right)}{\left(\Psi-\mu\right)\left(\Psi\varphi+2\mu\theta\right)}\right)\mathbb{A}_{t+1}$$

Having now $\mathbb{Y}_{t+1}$, from equations [33 - 35] and [43 - 44] we obtain $\hat{\tau}_{t+1}$ as a function of $\mathbb{C}_{t+1}$ and $\mathbb{A}_{t+1}$ only:

$$\hat{\tau}_{t+1}=-1/2\,\frac{\Psi\left(\rho\varphi\Psi+2\mu-\Psi\right)}{\left(\Psi-\mu\right)\left(\Psi\varphi+2\mu\theta\right)}\mathbb{C}_{t+1}+1/2\,\frac{\Psi^2\left(\varphi+1\right)}{\left(\Psi-\mu\right)\left(\Psi\varphi+2\mu\theta\right)}\mathbb{A}_{t+1} \tag{54}$$

Come back to (53) and replace $\mathbb{Y}_{t+1}$ and $\hat{\tau}_{t+1}$ to obtain

$$\begin{aligned}
\hat{W}_t &= \beta \hat{W}_{t+1} + \mathcal{A}\mathbb{C}_{t+1} + \mathcal{B}\mathbb{A}_{t+1} - \beta\hat{\xi}_{t+1}, \text{ with} \\
\mathcal{A} &= \frac{(-\mu\,\varphi + \Psi\,\varphi + \varphi\,\mu\,\theta\,\rho + \mu\,\theta - 1/2\,\rho\,\varphi\,\Psi - \mu + 1/2\,\Psi)\,\beta}{\Psi\,\varphi + 2\,\mu\,\theta} \\
\mathcal{B} &= -1/2\,\frac{(\varphi+1)\,(-\Psi + 2\,\mu\,\theta)\,\beta}{\Psi\,\varphi + 2\,\mu\,\theta}
\end{aligned} \tag{55}$$

Iterate (55) forward and take expectations at time $t+1$ to obtain:

$$\hat{W}_{t+1} = \mathcal{A}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{C}_{t+2+i}\right] + \mathcal{B}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{A}_{t+2+i}\right]$$

The last term dissapears, since $\hat{\xi}_{t+1}$ is a iid variable. To sumarize, we know that

$$\begin{aligned}
\hat{W}_t &= \beta \hat{W}_{t+1} + \mathcal{A}\mathbb{C}_{t+1} + \mathcal{B}\mathbb{A}_{t+1} - \beta\hat{\xi}_{t+1} \\
\hat{W}_{t+1} &= \mathcal{A}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{C}_{t+2+i}\right] + \mathcal{B}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{A}_{t+2+i}\right]
\end{aligned}$$

Replace $\hat{W}_{t+1}$ from second equation in first equation to obtain:

$$\begin{aligned}
\hat{W}_t = \; & \mathcal{A}\mathbb{C}_{t+1} + \beta\mathcal{A}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{C}_{t+2+i}\right] \\
& + \mathcal{B}\mathbb{A}_{t+1} + \beta\mathcal{B}\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\mathbb{A}_{t+2+i}\right] \\
& - \beta\hat{\xi}_{t+1}
\end{aligned} \tag{56}$$

Compute the right hand side term by term. From equation [29], we know that

$$\mathbb{C}_t = \mathbb{E}_t\left[\mathbb{C}_{t+1}\right] - \frac{1}{\rho}\mathbb{E}_t\left[\hat{Q}_{t+1} - \hat{Q}_t\right]$$

Iterate forward up to time $t+n$ to obtain.

$$\mathbb{C}_t - \frac{1}{\rho}\hat{Q}_t = \mathbb{E}_t\left[\mathbb{C}_{t+n} - \frac{1}{\rho}\hat{Q}_{t+n}\right]$$

Replace $\hat{Q}_t$ with $\left(\frac{2\mu}{\Psi} - 1\right)\hat{\tau}_t$, use (54), and write everything in terms of $\mathbb{C}_t$ and $\mathbb{A}_t$:

$$\begin{aligned}
\mathcal{C}\mathbb{C}_t + \mathcal{D}\mathbb{A}_t &= \mathcal{C}\mathbb{E}_t\left[\mathbb{C}_{t+n}\right] + \mathcal{D}\mathbb{E}_t\left[\mathbb{A}_{t+n}\right], \text{ with} \\
\mathcal{C} &= 1 + 1/2\left(2\frac{\mu}{\Psi} - 1\right)\frac{\Psi\left(\rho\varphi\Psi + 2\mu - \Psi\right)}{\rho\left(\Psi - \mu\right)\left(\Psi\varphi + 2\mu\theta\right)} \\
\mathcal{D} &= 1/2\frac{\left(-2\mu + \Psi\right)\Psi\left(\varphi + 1\right)}{\rho\left(\Psi - \mu\right)\left(\Psi\varphi + 2\mu\theta\right)}
\end{aligned}$$

We know that $\mathbb{E}_{t+1}\left[\mathbb{A}_{t+2+i}\right] = \zeta_A^{i+1}\mathbb{A}_{t+1}$. It follows that

$$\mathbb{E}_{t+1}\left[\mathbb{C}_{t+2+i}\right] = \mathbb{C}_{t+1} + \frac{\mathcal{D}}{\mathcal{C}}\mathbb{A}_{t+1} - \frac{\mathcal{D}}{\mathcal{C}}\zeta_A^{i+1}\mathbb{A}_{t+1} \tag{57}$$

We can replace this into the first line in (56):

$$\begin{aligned}
\mathcal{A}\mathbb{C}_{t+1}+\beta\mathcal{A}\sum_{i=0}^{\infty}\beta^{i}\mathbb{E}_{t+1}\left[\mathbb{C}_{t+2+i}\right] &= \mathcal{A}\mathbb{C}_{t+1}+\beta\mathcal{A}\sum_{i=0}^{\infty}\beta^{i}\left[\mathbb{C}_{t+1}+\frac{\mathcal{D}}{\mathcal{C}}\mathbb{A}_{t+1}-\frac{\mathcal{D}}{\mathcal{C}}\zeta_A^{i+1}\mathbb{A}_{t+1}\right] \\
&= \left(\mathcal{A}+\beta\mathcal{A}\frac{1}{1-\beta}\right)\mathbb{C}_{t+1}+\left(\beta\mathcal{A}\frac{\mathcal{D}}{\mathcal{C}}\frac{1}{1-\beta}-\mathcal{A}\frac{\mathcal{D}}{\mathcal{C}}\frac{\zeta_A}{1-\beta\zeta_A}\right)\mathbb{A}_{t+1}
\end{aligned}$$

Second line in (56) is

$$\begin{aligned}
\mathcal{B}\mathbb{A}_{t+1}+\beta\mathcal{B}\sum_{i=0}^{\infty}\beta^{i}\mathbb{E}_{t+1}\left[\mathbb{A}_{t+2+i}\right] &= \mathcal{B}\mathbb{A}_{t+1}+\mathcal{B}\mathbb{A}_{t+1}\sum_{i=1}^{\infty}\left(\beta\zeta_A\right)^{i} \\
&= \frac{\mathcal{B}}{1-\beta\zeta_A}\mathbb{A}_{t+1}
\end{aligned}$$

Finally, the equation (56) becomes:

$$\begin{aligned}
\hat{W}_t &= \left(\mathcal{A}+\beta\mathcal{A}\frac{1}{1-\beta}\right)\mathbb{C}_{t+1}+\left(\beta\mathcal{A}\frac{\mathcal{D}}{\mathcal{C}}\frac{1}{1-\beta}-\mathcal{A}\frac{\mathcal{D}}{\mathcal{C}}\frac{\zeta_A}{1-\beta\zeta_A}+\frac{\mathcal{B}}{1-\beta\zeta_A}\right)\mathbb{A}_{t+1}-\beta\hat{\xi}_{t+1} \\
&= \mathcal{E}\mathbb{C}_{t+1}+\mathcal{F}\mathbb{A}_{t+1}-\beta\hat{\xi}_{t+1}
\end{aligned}$$

with

$$\begin{aligned}
\mathcal{E} &= -1/2\frac{\left(2\theta\varphi\mu\rho+2\mu\theta-2\varphi\mu+2\Psi\varphi+\Psi-\rho\varphi\Psi-2\mu\right)\beta}{\left(\Psi\varphi+2\mu\theta\right)\left(-1+\beta\right)} \\
\mathcal{F} &= 1/4\beta^{2}\left(\left(2\varphi\mu\rho+2\mu\right)\theta-2\varphi\mu+2\Psi\varphi+\Psi-\rho\varphi\Psi-2\mu\right)\left(-2\mu+\Psi\right)\Psi\left(\varphi+1\right) \\
&\quad \left(\left(1-\beta\right)^{-1}-\frac{\zeta_A}{1-\beta\zeta_A}\right)\left(\Psi\varphi+2\mu\theta\right)^{-2}\rho^{-1}\left(\Psi-\mu\right)^{-1} \\
&\quad \left(1+1/2\left(2\frac{\mu}{\Psi}-1\right)\Psi\left(\rho\varphi\Psi+2\mu-\Psi\right)\rho^{-1}\left(\Psi-\mu\right)^{-1}\left(\Psi\varphi+2\mu\theta\right)^{-1}\right)^{-1} \\
&\quad -1/2\frac{\left(\varphi+1\right)\left(-\Psi+2\mu\theta\right)\beta}{\left(\Psi\varphi+2\mu\theta\right)\left(1-\beta\zeta_A\right)}
\end{aligned}$$

From here we obtain $\mathbb{C}_{t+1}$:

$$\mathbb{C}_{t+1}=\frac{1}{\mathcal{E}}\hat{W}_t-\frac{\mathcal{F}}{\mathcal{E}}\mathbb{A}_{t+1}+\frac{\beta}{\mathcal{E}}\hat{\xi}_{t+1} \tag{58}$$

The term that we need to compute is $\hat{C}_{t+1}-\hat{C}^{*}_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}$:

$$\hat{C}_{t+1}-\hat{C}^{*}_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}=\mathbb{C}_{t+1}-\frac{1}{\rho}\left(\frac{2\mu}{\Psi}-1\right)\hat{\tau}_{t+1}$$

Use (54) to replace $\hat{\tau}_{t+1}$ and obtain

$$\mathbb{C}_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}=\mathcal{J}\hat{W}_t+\mathcal{K}\mathbb{A}_{t+1}+\mathcal{L}\hat{\xi}_{t+1} \tag{59}$$

with

$$\begin{aligned}\mathcal{J} &= \frac{(1-\beta)\left(\rho\Psi^2\varphi+4\rho\Psi\mu\theta-4\rho\mu^2\theta+4\mu^2-4\mu\Psi+\Psi^2\right)}{\beta\left(2\theta\varphi\mu\rho+2\mu\theta-2\varphi\mu+2\Psi\varphi+\Psi-\rho\varphi\Psi-2\mu\right)(\Psi-\mu)\rho} \\ \mathcal{K} &= \frac{(\varphi+1)(-1+\beta)(2\rho\mu\theta-2\mu+\Psi-\Psi\rho)}{\rho\left(2\theta\varphi\mu\rho+2\mu\theta-2\varphi\mu+2\Psi\varphi+\Psi-\rho\varphi\Psi-2\mu\right)(-1+\beta\zeta_A)} \\ \mathcal{L} &= \frac{(1-\beta)\left(\rho\Psi^2\varphi+4\rho\Psi\mu\theta-4\rho\mu^2\theta+4\mu^2-4\mu\Psi+\Psi^2\right)}{\left(2\theta\varphi\mu\rho+2\mu\theta-2\varphi\mu+2\Psi\varphi+\Psi-\rho\varphi\Psi-2\mu\right)(\Psi-\mu)\rho}\end{aligned}$$

Second Term $(\hat{r}_{x,t+1})$

From equations [38] and [47] we obtain

$$\hat{r}_{x,t+1} = (1-\beta)\,\mathbb{P}_{t+1} + \beta\mathbb{V}_{t+1} - \mathbb{V}_t - \left(\frac{2\mu}{\Psi}-1\right)(\hat{\tau}_{t+1}-\hat{\tau}_t) \tag{60}$$

where $\mathbb{P}_{t+1} = \hat{\pi}_{t+1}(h) - \hat{\pi}^*_{t+1}(f)$ and $\mathbb{V}_{t+1} = \hat{v}_{t+1} - \hat{v}^*_{t+1}$. We know that $\mathbb{E}_t[\hat{r}_{x,t+1}] = 0$. Thus

$$0 = (1-\beta)\,\mathbb{E}_t[\mathbb{P}_{t+1}] + \beta\mathbb{E}_t[\mathbb{V}_{t+1}] - \mathbb{V}_t - \left(\frac{2\mu}{\Psi}-1\right)\mathbb{E}_t[\hat{\tau}_{t+1}] + \left(\frac{2\mu}{\Psi}-1\right)\hat{\tau}_t$$

or

$$-\mathbb{V}_t + \left(\frac{2\mu}{\Psi}-1\right)\hat{\tau}_t = -(1-\beta)\,\mathbb{E}_t[\mathbb{P}_{t+1}] - \beta\mathbb{E}_t[\mathbb{V}_{t+1}] + \left(\frac{2\mu}{\Psi}-1\right)\mathbb{E}_t[\hat{\tau}_{t+1}]$$

Replace it in (60) to obtain

$$\hat{r}_{x,t+1} = (1-\beta)(\mathbb{P}_{t+1}-\mathbb{E}_t[\mathbb{P}_{t+1}]) + \beta(\mathbb{V}_{t+1}-\mathbb{E}_t[\mathbb{V}_{t+1}]) - \left(\frac{2\mu}{\Psi}-1\right)(\hat{\tau}_{t+1}-\mathbb{E}_t[\hat{\tau}_{t+1}]) \tag{61}$$

We will compute (61) term by term. From [36] and [45] we obtain:

$$\mathbb{P}_{t+1} = \mathbb{Y}_{t+1} + 2\left(\frac{\mu}{\Psi}-1\right)\hat{\tau}_{t+1}$$

Both $\mathbb{Y}_{t+1}$ and $\hat{\tau}_{t+1}$ are known from the previous point, as function of $\mathbb{C}_{t+1}$ and $\mathbb{A}_{t+1}$. Substituting them into the last equation yields

$$\begin{aligned}\mathbb{P}_{t+1} &= \mathcal{M}\mathbb{C}_{t+1} + \mathcal{N}\mathbb{A}_{t+1}, \text{ with} \\ \mathcal{M} &= \frac{2\mu\varphi-\Psi\varphi-2\varphi\mu\theta\rho+\rho\varphi\Psi+2\mu-\Psi}{\Psi\varphi+2\mu\theta} \\ \mathcal{N} &= \frac{(-\Psi+2\mu\theta)(\varphi+1)}{\Psi\varphi+2\mu\theta}\end{aligned} \tag{62}$$

We can substitute $\mathbb{C}_{t+1}$ from (58):

$$\begin{aligned}\mathbb{P}_{t+1} &= \mathcal{M}\left(\frac{1}{\mathcal{E}}\hat{W}_t - \frac{\mathcal{F}}{\mathcal{E}}\mathbb{A}_{t+1} + \frac{\beta}{\mathcal{E}}\hat{\xi}_{t+1}\right) + \mathcal{N}\mathbb{A}_{t+1} \\ &= \frac{\mathcal{M}}{\mathcal{E}}\hat{W}_t + \left(\mathcal{N}-\frac{\mathcal{M}\mathcal{F}}{\mathcal{E}}\right)\mathbb{A}_{t+1} + \frac{\mathcal{M}\beta}{\mathcal{E}}\hat{\xi}_{t+1}\end{aligned}$$

Then $\mathbb{P}_{t+1} - \mathbb{E}_t[\mathbb{P}_{t+1}]$ will be equal to

$$\mathbb{P}_{t+1} - \mathbb{E}_t[\mathbb{P}_{t+1}] = \left(\mathcal{N}-\frac{\mathcal{M}\mathcal{F}}{\mathcal{E}}\right)\mathbb{A}_{t+1} + \frac{\mathcal{M}\beta}{\mathcal{E}}\hat{\xi}_{t+1} \tag{63}$$

Note that I ignore terms at time t since at the end, they will all reduce to 0. Second term to be computed is $\mathbb{V}_{t+1} - \mathbb{E}_t\left[\mathbb{V}_{t+1}\right]$. The Euler equations for home and foreign country are:

$$\begin{aligned}
(C_t)^{-\rho} &= \beta\mathbb{E}_t\left[(C_{t+1})^{-\rho}\,\frac{\pi_{t+1}+v_{t+1}}{v_t}\right] \\
(C_t^*)^{-\rho} &= \beta\mathbb{E}_t\left[(C_{t+1}^*)^{-\rho}\,\frac{\pi_{t+1}^*+v_{t+1}^*}{v_t^*}\right]
\end{aligned}$$

Bring v_t and v_t^* on the left hand side. Iterate forward, impose transversality conditions and obtain

$$\begin{aligned}
v_t &= \sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\left(\frac{C_{t+i}}{C_t}\right)^{-\rho}\pi_{t+i}\right] \\
v_t^* &= \sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\left(\frac{C_{t+i}^*}{C_t^*}\right)^{-\rho}\pi_{t+i}^*\right]
\end{aligned}$$

At the steady state we have $v = \frac{\beta}{1-\beta}\pi$. Log-linearize around the steady state:

$$\begin{aligned}
\frac{\beta}{1-\beta}\hat{v}_t &= \rho\frac{\beta}{1-\beta}\hat{C}_t - \rho\sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\hat{C}_{t+i}\right] + \sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\hat{\pi}_{t+i}\right] \\
\frac{\beta}{1-\beta}\hat{v}_t^* &= \rho\frac{\beta}{1-\beta}\hat{C}_t^* - \rho\sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\hat{C}_{t+i}^*\right] + \sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\hat{\pi}_{t+i}^*\right]
\end{aligned}$$

Take the difference of these equations and write everything at time $t+1$:

$$\mathbb{V}_{t+1} = \rho\mathbb{C}_{t+1} - \rho\frac{1-\beta}{\beta}\sum_{i=1}^{\infty}\beta^i\mathbb{E}_{t+1}\left[\mathbb{C}_{t+1+i}\right] + \frac{1-\beta}{\beta}\sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\mathbb{P}_{t+1+i}\right] \tag{64}$$

Following (57) and (62) we have

$$\sum_{i=1}^{\infty}\beta^i\mathbb{E}_{t+1}\left[\mathbb{C}_{t+1+i}\right] = \frac{\beta}{1-\beta}\mathbb{C}_{t+1} + \left(\frac{\mathcal{D}}{\mathcal{C}}\frac{\beta}{1-\beta} - \frac{\mathcal{D}}{\mathcal{C}}\frac{\beta\zeta_A}{1-\beta\zeta_A}\right)\mathbb{A}_{t+1}$$

and

$$\sum_{i=1}^{\infty}\beta^i\mathbb{E}_t\left[\mathbb{P}_{t+1+i}\right] = \frac{\mathcal{M}\beta}{1-\beta}\mathbb{C}_{t+1} + \left[\frac{\mathcal{M}\mathcal{D}}{\mathcal{C}}\frac{\beta}{1-\beta} + \left(\mathcal{N} - \frac{\mathcal{M}\mathcal{D}}{\mathcal{C}}\right)\frac{\beta\zeta_A}{1-\beta\zeta_A}\right]\mathbb{A}_{t+1}$$

Regroup everything into (64):

$$\mathbb{V}_{t+1} = \mathcal{P}\mathbb{C}_{t+1} + \mathcal{Q}\mathbb{A}_{t+1}$$

with $\mathcal{P}$ and $\mathcal{Q}$ higly complicated expressions which I do not show here to save space. Then $\mathbb{V}_{t+1} - \mathbb{E}_t\left[\mathbb{V}_{t+1}\right]$ will be equal to

$$\mathbb{V}_{t+1} - \mathbb{E}_t\left[\mathbb{V}_{t+1}\right] = \left(\mathcal{Q} - \frac{\mathcal{P}\mathcal{F}}{\mathcal{E}}\right)\mathbb{A}_{t+1} + \frac{\mathcal{P}\beta}{\mathcal{E}}\hat{\xi}_{t+1} \tag{65}$$

Finnally, last term in (61) to be comuted is $\hat{\tau}_{t+1} - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]$. Use formula (54) to obtain

$$
\begin{aligned}
\hat{\tau}_{t+1} &= \mathcal{R}\mathbb{C}_{t+1} + \mathcal{S}\mathbb{A}_{t+1}, \text{ with} \\
\mathcal{R} &= -1/2\,\frac{\Psi\left(\rho\varphi\Psi + 2\mu - \Psi\right)}{\left(\Psi - \mu\right)\left(\Psi\varphi + 2\mu\theta\right)} \\
S &= 1/2\,\frac{\Psi^2\left(\varphi + 1\right)}{\left(\Psi - \mu\right)\left(\Psi\varphi + 2\mu\theta\right)}
\end{aligned}
$$

and

$$
\hat{\tau}_{t+1} - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right] = \left(\mathcal{S} - \frac{\mathcal{R}\mathcal{F}}{\mathcal{E}}\right)\mathbb{A}_{t+1} + \frac{\mathcal{R}\beta}{\mathcal{E}}\hat{\xi}_{t+1} \tag{66}
$$

Regroup now (63), (65) and (66) into (61):

$$
\hat{r}_{x,t+1} = (1-\beta)\left(\mathbb{P}_{t+1} - \mathbb{E}_t\left[\mathbb{P}_{t+1}\right]\right) + \beta\left(\mathbb{V}_{t+1} - \mathbb{E}_t\left[\mathbb{V}_{t+1}\right]\right) - \left(\frac{2\mu}{\Psi} - 1\right)\left(\hat{\tau}_{t+1} - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) \tag{67}
$$

$$
\begin{aligned}
\hat{r}_{x,t+1} &= \mathcal{T}\mathbb{A}_{t+1} + \mathcal{U}\hat{\xi}_{t+1}, \text{ with} \\
\mathcal{T} &= 2\,\frac{\left(\varphi+1\right)\left(-1+\beta\right)\mu\left(\theta-1\right)}{\left(-2\mu\varphi + 2\Psi\varphi + 2\varphi\mu\theta\rho + 2\mu\theta - \rho\varphi\Psi - 2\mu + \Psi\right)\left(-1+\beta\zeta_A\right)} \\
\mathcal{U} &= \frac{\left(-1+\beta\right)\left(\Psi^2 - 4\varphi\mu^2\theta\rho - \rho\Psi^2\varphi - 6\mu\varphi\Psi + 4\varphi\mu\theta\rho\Psi - 2\mu\Psi + 4\mu^2\varphi + 2\Psi^2\varphi\right)}{\left(\Psi - \mu\right)\left(-2\mu\varphi + 2\Psi\varphi + 2\varphi\mu\theta\rho + 2\mu\theta - \rho\varphi\Psi - 2\mu + \Psi\right)}
\end{aligned}
$$

Solution for $\tilde{\alpha}$

We can use now the closed form solution in Devereux and Sutherland (2007) to obtain

$$
\tilde{\alpha} = \frac{\left(2\mu\theta\rho - 2\mu + \Psi - \rho\Psi\right)\left(\Psi - \mu\right)}{\left(-1+\beta\right)\left(4\rho\Psi\mu\theta - 4\rho\mu^2\theta + 4\mu^2 - 4\mu\Psi - \rho\Psi^2 + \Psi^2\right)}
$$

Compute then the proportion of home equity held by home residents:

$$
\begin{aligned}
\alpha_t^E &= \frac{\alpha_{E,t} + N_D v_t}{N_D v_t} = \frac{\tilde{\alpha}\beta N_D Y\left(h\right)\frac{p(h)}{P} + N_D\frac{\beta}{1-\beta}\pi}{N_D\frac{\beta}{1-\beta}\pi} = 1 + \frac{\tilde{\alpha}N_D Y\left(h\right)\frac{p(h)}{P}}{N_D\frac{1}{1-\beta}\frac{1}{\phi}N_D Y\left(h\right)\frac{p(h)}{P}} \\
&= 1 + \frac{\tilde{\alpha}}{\frac{1}{1-\beta}\frac{1}{\phi}} = 1 + \tilde{\alpha}\phi\left(1-\beta\right)
\end{aligned}
$$

In the benchmark case the share of capital income in total income is defined as

$$
\frac{\pi}{N_D\tilde{Y}\frac{\tilde{p}(h)}{P}} = \frac{1}{\phi}
$$

It then follows that the share of labor income Λ is $1 - \frac{1}{\Upsilon\phi}$. The benchmark case closed form solution becomes

$$
1 - \frac{\left(2\theta\mu\rho - 2\mu - \rho\Psi + \Psi\right)\left(-\Psi + \mu\right)\phi}{\left(4\rho\mu^2\theta - 4\mu^2 - 4\rho\Psi\mu\theta + 4\mu\Psi + \rho\Psi^2 - \Psi^2\right)} = 1 - \frac{\left(2\theta\mu\rho - 2\mu - \rho\Psi + \Psi\right)\left(-\Psi + \mu\right)}{\Gamma}\phi
$$

with $\Gamma = \rho\Psi^2 - 4\rho\Psi\mu\theta + 4\rho\mu^2\theta - \Psi^2 + 4\mu\Psi - 4\mu^2$. Rearrange terms to obtain

$$
\begin{aligned}
& \frac{1}{2}+\frac{1}{2}+\frac{(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi)}{\Gamma}(\phi-1)+\frac{(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi)}{\Gamma} \\
= & \frac{1}{2}+\frac{\Gamma+2(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi)}{2\Gamma}+\frac{\Lambda}{1-\Lambda}\frac{(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi)}{\Gamma}
\end{aligned}
$$

Take the numerator of the second term:

$$
\begin{aligned}
\Gamma+2(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi) &= -\rho\Psi^2+\Psi^2-2\mu\Psi+2\rho\Psi\mu \\
&= \rho\left(2\Psi\mu-\Psi^2\right)-\left(2\Psi\mu-\Psi^2\right) \\
&= (\rho-1)\Psi(2\mu-\Psi)
\end{aligned}
$$

This leads to the following solution for α_t^E:

$$
\alpha_t^E=\frac{1}{2}+(\rho-1)\frac{1}{2}\frac{\Psi(2\mu-\Psi)}{\Gamma}+\frac{\Lambda}{1-\Lambda}\frac{(\Psi-\mu)(2\rho\theta\mu-2\mu+\Psi-\rho\Psi)}{\Gamma}
$$

Appendix E

Proof that $\Psi\left(2\mu-\Psi\right)$ is between 0 and 1

We know that $\Psi=\mu+\left(1-\mu\right)t^{1-\theta}$, thus we will have

$$\mu\leq\Psi\leq1$$

and

$$2\mu-1\leq2\mu-\Psi\leq\mu$$

Multiplying these two equations yields

$$0\leq\mu\left(2\mu-1\right)\leq\Psi\left(2\mu-\Psi\right)\leq\mu\leq1$$

The function $\Psi\left(2\mu-\Psi\right)$ is an increasing function in μ and t:

$$\begin{aligned}\frac{\partial}{\partial\mu}\Psi\left(2\mu-\Psi\right) &= 2\mu+\left(2\mu-1\right)t^{1-\theta}>0\\ \frac{\partial}{\partial t}\Psi\left(2\mu-\Psi\right) &= -\frac{\mu}{1-\mu}t^{1-\theta}\left(1-\theta\right)>0\end{aligned}$$

Proof that $\left(\Psi-\mu\right)\left(2\rho\theta\mu-2\mu+\Psi-\rho\Psi\right)$ is positive

First term is positive, since $\Psi\geq\mu$. We use the same inequality for the second term:

$$2\rho\theta\mu-2\mu+\Psi\left(1-\rho\right)\geq2\rho\theta\mu-2\mu+\mu\left(1-\rho\right)=\left[\rho\left(2\theta-1\right)-1\right]\mu>0$$

It then follows that

$$\left(\Psi-\mu\right)\left(2\rho\theta\mu-2\mu+\Psi-\rho\Psi\right)\geq0$$

Appendix F

Model with heterogeneous firms and export costs

$$W_t = \alpha_{E,t-1} r_{E,t} + \alpha_{E^*,t-1} r^*_{E,t} + N_D \tilde{Y}_t \frac{\tilde{p}_t(h)}{P_t} - C_t \tag{68}$$

$$N_D^{\frac{\theta-\phi}{1-\phi}} \tilde{Y}_{D,t} \left(\frac{\tilde{p}_t(h)}{P_t} \right)^{\theta} = \mu C_t \tag{69}$$

$$N_X^{\frac{\theta-\phi}{1-\phi}} \tilde{Y}_{X,t} \left(\frac{\tilde{p}^*_t(h)}{P^*_t} \right)^{\theta} \frac{1}{t} = (1-\mu) C^*_t \tag{70}$$

$$\tilde{Y}_t = \tilde{Y}_{D,t} + \left(\frac{1}{\eta} \right)^{k+1} \tilde{Y}_{X,t} \tag{71}$$

$$\frac{\tilde{p}_t(h)}{P_t} = \frac{\phi}{\phi-1} \frac{w_t}{\tilde{z}_D A_t} \tag{72}$$

$$N_D Y_t = A_t \tilde{z}_D l_t \tag{73}$$

$$L_t = l_t + N_X \frac{f_X}{A_t} \tag{74}$$

$$w_t = \kappa C_t^{\rho} L_t^{\frac{1}{\varphi}} \tag{75}$$

$$\tilde{\pi}_t + w_t \chi \frac{f_X}{A_t} = \frac{1}{\phi} \tilde{Y}_t \frac{\tilde{p}_t(h)}{P_t} \tag{76}$$

$$C_t^{-\rho} = \mathbb{E}_t \left[C_{t+1}^{-\rho} r_{E,t+1} \right] \tag{77}$$

$$C_t^{-\rho} = \mathbb{E}_t \left[C_{t+1}^{-\rho} r^*_{E,t+1} \right] \tag{78}$$

$$r_{E,t} = \frac{\tilde{\pi}_t + \tilde{v}_t}{\tilde{v}_{t-1}} \tag{79}$$

and equations (69) - (79) for the foreign country:

$$(N^*_D)^{\frac{\theta-\phi}{1-\phi}} \tilde{Y}^*_{D,t} \left(\frac{\tilde{p}^*_t(f)}{P^*_t} \right)^{\theta} = \mu C^*_t \tag{80}$$

$$(N^*_X)^{\frac{\theta-\phi}{1-\phi}} \tilde{Y}^*_{X,t} \left(\frac{\tilde{p}_t(f)}{P_t} \right)^{\theta} \frac{1}{t} = (1-\mu) C_t \tag{81}$$

$$\tilde{Y}^*_t = \tilde{Y}^*_{D,t} + \left(\frac{1}{\eta} \right)^{k+1} \tilde{Y}^*_{X,t} \tag{82}$$

$$\frac{\tilde{p}^*_t(f)}{P^*_t} = \frac{\phi}{\phi-1} \frac{w^*_t}{\tilde{z}_D A^*_t} \tag{83}$$

$$N^*_D Y^*_t = A^*_t \tilde{z}_D l^*_t \tag{84}$$

$$L^*_t = l^*_t + N^*_X \frac{f_X}{A^*_t} \tag{85}$$

$$w^*_t = \kappa (C^*_t)^{\rho} (L^*_t)^{\frac{1}{\varphi}} \tag{86}$$

$$\tilde{\pi}^*_t + w^*_t \chi \frac{f_X}{A^*_t} = \frac{1}{\phi} \tilde{Y}^*_t \frac{\tilde{p}^*_t(f)}{P^*_t} \tag{87}$$

$$(C^*_t)^{-\rho} = \mathbb{E}_t \left[\frac{Q_t}{Q_{t+1}} (C^*_{t+1})^{-\rho} r_{E,t+1} \right] \tag{88}$$

$$(C^*_t)^{-\rho} = \mathbb{E}_t \left[\frac{Q_t}{Q_{t+1}} (C^*_{t+1})^{-\rho} r^*_{E,t+1} \right] \tag{89}$$

$$r_{E,t} = \frac{\tilde{\pi}^*_t + \tilde{v}^*_t}{\tilde{v}^*_{t-1}} \frac{Q_{t-1}}{Q_t} \tag{90}$$

Steady-state calculation

Variables without time subscript represent steady-state values. The aggregate consumer price index is

$$\begin{aligned} P_t^{1-\theta} &= \left\{ \mu P_{H,t}^{1-\theta} + (1-\mu) P_{F,t}^{1-\theta} \right\} \\ &= N_D^{\frac{1-\theta}{1-\phi}} \mu \left[p_t(h) \right]^{1-\theta} + N_X^{\frac{1-\theta}{1-\phi}} (1-\mu) \left[p_t(f) \right]^{1-\theta} \end{aligned}$$

Divide by $[p_t(h)]^{1-\theta}$

$$\begin{aligned} \left[\frac{P_t}{p_t(h)} \right]^{1-\theta} &= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) \chi^{\frac{1-\theta}{1-\phi}} \left[\frac{p_t(f)}{p_t(h)} \right]^{1-\theta} \right\} \\ &= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) \chi^{\frac{1-\theta}{1-\phi}} \left(\frac{t}{\eta} \right)^{1-\theta} \left[\frac{p_t^*(f)}{p_t(h)} \right]^{1-\theta} \right\} \end{aligned}$$

In a symmetrical steady state we have $\frac{p^*(f)}{p(h)} = 1$, thus we obtain

$$\begin{aligned} \left[\frac{P}{p(h)} \right]^{1-\theta} &= N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) \chi^{\frac{1-\theta}{1-\phi}} \left(\frac{t}{\eta} \right)^{1-\theta} \right\} \\ &= N_D^{\frac{1-\theta}{1-\phi}} \Theta \end{aligned}$$

with $\Theta = \left\{ \mu + (1-\mu) \chi^{\frac{1-\theta}{1-\phi}} \left(\frac{t}{\eta} \right)^{1-\theta} \right\}$. We obtain therefore

$$\frac{p(h)}{P} = N_D^{\frac{1}{\phi-1}} \Theta^{\frac{1}{\theta-1}} \tag{91}$$

I solve first for the fixed export cost f_X which... completeaza aici. The average profit from exporting is:

$$\tilde{\pi}_X = \frac{1}{\phi\eta} \tilde{Y}_X \frac{\tilde{p}(h)}{P} - \frac{w f_X}{A} \tag{92}$$

Proof Start with total revenues from exporting for all the N_X firms:

$$N_X \tilde{Y}_{X,t} \frac{\tilde{p}_t^*(h)}{P_t^*} Q_t \frac{1}{t} = N_X \tilde{\pi}_{X,t} + N_X w_t \tilde{l}_{X,t} + N_X \frac{w_t f_X}{A_t}, \tag{93}$$

where $\tilde{l}_{X,t}$ represents the average labor used by each exporting firm. Thus, we know that

$$\tilde{l}_{X,t} = \frac{\tilde{Y}_{X,t}}{\tilde{z}_X A_t} = \frac{1}{\eta} \frac{\tilde{Y}_{X,t}}{\tilde{z}_D A_t}$$

Replacing this in (93) and simplifying N_X yields

$$\begin{aligned} \tilde{Y}_{X,t} \frac{\tilde{p}_t^*(h)}{P_t^*} Q_t \frac{1}{t} &= \tilde{\pi}_{X,t} + \frac{w_t}{\tilde{z}_D A_t} \frac{1}{\eta} \tilde{Y}_{X,t} + \frac{w_t f_X}{A_t} \\ &= \tilde{\pi}_{X,t} + \frac{\phi-1}{\phi} \frac{\tilde{p}_t(h)}{P_t} \frac{1}{\eta} \tilde{Y}_{X,t} + \frac{w_t f_X}{A_t} \end{aligned}$$

We know that $\frac{\tilde{p}_t^*(h)}{P_t^*} = \frac{1}{Q_t} \frac{t}{\eta} \frac{\tilde{p}_t(h)}{P_t}$. Substitute this in the lase equation to obtain

$$\frac{1}{\eta} \tilde{Y}_{X,t} \frac{\tilde{p}_t(h)}{P_t} = \tilde{\pi}_{X,t} + \frac{\phi-1}{\phi} \frac{1}{\eta} \tilde{Y}_{X,t} \frac{\tilde{p}_t(h)}{P_t} + \frac{w_t f_X}{A_t},$$

and, finally

$$\tilde{\pi}_{X,t} = \frac{1}{\phi\eta}\tilde{Y}_{X,t}\frac{\tilde{p}_t(h)}{P_t} - \frac{w_t f_X}{A_t}.$$

Evaluating this equation at the steady state, one obtains (92). We also know that, given the fixed costs f_X, there is a firm with productivity $z_X \geq z_{\min}$ such that it's exporting profits are zero. All firms with productivity larger than z_X will export and all firms with productivity between $z_{\min}$ and z_X will produce only for the home market. This is because for them exporting is not profitable. From the firm with productivity equal to the threshold z_X, the zero-export cutoff condition is

$$\pi_{X,t} = \frac{Q_t}{\phi}\left(\frac{\phi}{\phi-1}t\frac{1}{Q_t}\frac{w_t}{z_X A_t}\right)^{1-\theta} N_X^{\frac{\phi-\theta}{1-\phi}}(1-\mu)C_t^* - \frac{w_t f_X}{A_t} = 0 \tag{94}$$

For the exporting firms, the average profit is equal to

$$\tilde{\pi}_{X,t} = \frac{Q_t}{\phi}\left(\frac{\phi}{\phi-1}t\frac{1}{Q_t}\frac{w_t}{\tilde{z}_X A_t}\right)^{1-\theta} N_X^{\frac{\phi-\theta}{1-\phi}}(1-\mu)C_t^* - \frac{w_t f_X}{A_t}$$

We know that $\tilde{z}_X = z_X\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1}{\phi-1}}$. We replace this in the last equation to obtain

$$\begin{aligned}
\tilde{\pi}_{X,t} &= \left[\frac{k}{k-(\phi-1)}\right]^{\frac{1-\theta}{1-\phi}}\frac{Q_t}{\phi}\left(\frac{\phi}{\phi-1}t\frac{1}{Q_t}\frac{w_t}{z_X A_t}\right)^{1-\theta} N_X^{\frac{\phi-\theta}{1-\phi}}(1-\mu)C_t^* - \frac{w_t f_X}{A_t} \\
&= \left(\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1-\theta}{1-\phi}} - 1\right)\frac{w_t f_X}{A_t},
\end{aligned}$$

by using the zero export profit cutoff condition (94). Taking now this equation and combining it with (92) yields

$$\begin{aligned}
\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1-\theta}{1-\phi}}\frac{w_t f_X}{A_t} &= \frac{1}{\phi\eta}\tilde{Y}_X\frac{\tilde{p}(h)}{P} \\
&= \frac{1}{\phi}\frac{1}{\chi}\left(1-\frac{\mu}{\Theta}\right)\tilde{Y}\frac{\tilde{p}(h)}{P}
\end{aligned}$$

We know that $\frac{w_t}{\tilde{z}_D A_t} = \frac{\phi-1}{\phi}\frac{\tilde{p}_t(h)}{P_t}$. Thus, we can replace $\frac{w_t}{A_t}$:

$$\left[\frac{k}{k-(\phi-1)}\right]^{\frac{1-\theta}{1-\phi}}\tilde{z}_D\frac{\phi-1}{\phi}f_X\frac{\tilde{p}(h)}{P_t} = \frac{1}{\phi}\frac{1}{\chi}\left(1-\frac{\mu}{\Theta}\right)\tilde{Y}\frac{\tilde{p}(h)}{P}$$

Simplify by $\frac{\tilde{p}_t(h)}{P_t}$ to obtain the level of fixed export costs f_X which match the steady state fraction of exporters $\chi = \frac{N_X}{N_D}$:

$$f_X = \frac{1}{(\phi-1)\tilde{z}_D}\left[\frac{k-(\phi-1)}{k}\right]^{\frac{1-\theta}{1-\phi}}\frac{1}{\chi}\left(1-\frac{\mu}{\Theta}\right)\tilde{Y}$$

From here I will proceed as in the steady state calculation of the benchmark model. I fix $\tilde{Y} = 1$ and $\bar{A} = 1$. Then the steady state calculation reduces to the following system of equations:

$$\begin{cases}
N_D\tilde{Y}\frac{\tilde{p}(h)}{P} = C \\
\frac{\tilde{p}(h)}{P} = \frac{\phi}{\phi-1}\frac{w}{A\tilde{z}_D} \\
N_D\tilde{Y} = Al\tilde{z}_D \\
L = l + N_X\frac{f_X}{A} \\
w = \kappa C^{\rho}L^{\frac{1}{\varphi}}
\end{cases}$$

Solve now for N_D as in the benchmark model:

$$N_D = \left[\Theta^{\frac{\rho-1}{\theta-1}} \kappa \frac{\phi}{\phi-1} \tilde{Y}^{\rho} \left(\frac{\tilde{Y}}{\tilde{z}_D} + \chi f_X \right)^{\frac{1}{\varphi}} A^{-\frac{1}{\rho}} \frac{1}{A \tilde{z}_D} \right]^{\frac{1-\rho\phi}{\phi-1} - \frac{1}{\varphi}}$$

The other steady-state values follow.

Log-linearization

We need to re-define $\hat{p}_t(h) - \hat{P}_t, \hat{p}^*_t(h) - \hat{P}^*_t$, and $\hat{Q}_t$ in terms of variations of the terms of trade, which I define as $\hat{\tau}_t = \hat{p}^*_t(f) - \hat{p}_t(h)$. From the steady-state calculation, we know that

$$\left[\frac{P_t}{p_t(h)} \right]^{1-\theta} = N_D^{\frac{1-\theta}{1-\phi}} \left\{ \mu + (1-\mu) \chi^{\frac{1-\theta}{1-\phi}} \left(\frac{t}{\eta} \right)^{1-\theta} \left[\frac{p^*_t(f)}{p_t(h)} \right]^{1-\theta} \right\}$$

After some manipulations we obtain

$$\begin{aligned}
\hat{p}_t(h) - \hat{P}_t &= \left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t \\
\hat{p}_t(f) - \hat{P}_t &= \frac{\mu}{\Theta} \hat{\tau}_t \\
\hat{p}^*_t(f) - \hat{P}^*_t &= -\left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t \\
\hat{p}^*_t(h) - \hat{P}^*_t &= -\frac{\mu}{\Theta} \hat{\tau}_t \\
\hat{Q}_t &= \left(\frac{2\mu}{\Theta} - 1 \right) \hat{\tau}_t
\end{aligned}$$

As in Devereux and Sutherland (2007), I define $\hat{W}_t = \frac{W_t - W}{N_D \tilde{Y} \frac{\tilde{p}(h)}{P}}$, and $\tilde{\alpha} = \frac{\bar{\alpha}}{\beta N_D \tilde{Y} \frac{\tilde{p}(h)}{P}}$. With this notation, the log-linearization of the model equations becomes

$$\hat{W}_t = \frac{1}{\beta} \hat{W}_{t-1} + \hat{Y}_t + \left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t - \hat{C}_t + \tilde{\alpha} \hat{r}_{x,t} \tag{95}$$

$$\rho \hat{C}_t - \rho \mathbb{E}_t \left[\hat{C}_{t+1} \right] = \rho \hat{C}^*_t - \rho \mathbb{E}_t \left[\hat{C}^*_{t+1} \right] + \left(\frac{2\mu}{\Theta} - 1 \right) (\hat{\tau}_t - \mathbb{E}_t [\hat{\tau}_{t+1}]) \tag{96}$$

$$\hat{Y}_{D,t} + \theta \left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t = \hat{C}_t \tag{97}$$

$$\hat{Y}_{X,t} - \theta \frac{\mu}{\Theta} \hat{\tau}_t = \hat{C}^*_t \tag{98}$$

$$\hat{Y}_t = \frac{\mu}{\Theta} \hat{Y}_{D,t} + \left(1 - \frac{\mu}{\Theta} \right) \hat{Y}_{X,t} \tag{99}$$

$$\left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t = \hat{w}_t - \hat{A}_t \tag{100}$$

$$\hat{Y}_t = \hat{A}_t + \hat{l}_t \tag{101}$$

$$L \hat{L}_t = l \hat{l}_t + \frac{N_X f_X}{A} \hat{A}_t \tag{102}$$

$$\hat{w}_t = \rho \hat{C}_t + \frac{1}{\varphi} \hat{L}_t \tag{103}$$

$$\pi \hat{\pi}_t + w \chi \frac{f_X}{A} \left(\hat{w}_t - \hat{A}_t \right) = \frac{1}{\phi} \tilde{Y} \frac{\tilde{p}(h)}{P} \left[\hat{Y}_t + \left(\frac{\mu}{\Theta} - 1 \right) \hat{\tau}_t \right] \tag{104}$$

$$(-\rho) \hat{C}_t = -\rho \mathbb{E}_t \left[\hat{C}_{t+1} \right] + \mathbb{E}_t [\hat{r}_{E,t+1}] \tag{105}$$

$$\hat{r}_{E,t} = (1-\beta) \hat{\pi}_t + \beta \hat{v}_t - \hat{v}_{t-1} \tag{106}$$

Add the equations corresponding to the foreign country:

$$\hat{Y}^*_{D,t} - \theta\left(\frac{\mu}{\Theta} - 1\right)\hat{\tau}_t = \hat{C}^*_t \tag{107}$$

$$\hat{Y}^*_{X,t} + \theta\frac{\mu}{\Theta}\hat{\tau}_t = \hat{C}_t \tag{108}$$

$$\hat{Y}^*_t = \frac{\mu}{\Theta}\hat{Y}^*_{D,t} + \left(1 - \frac{\mu}{\Theta}\right)\hat{Y}^*_{X,t} \tag{109}$$

$$\left(1 - \frac{\mu}{\Theta}\right)\hat{\tau}_t = \hat{w}^*_t - \hat{A}^*_t \tag{110}$$

$$\hat{Y}^*_t = \hat{A}^*_t + \hat{l}^*_t \tag{111}$$

$$L^*\hat{L}^*_t = l^*\hat{l}^*_t + \frac{N^*_X f_X}{A^*}\hat{A}^*_t \tag{112}$$

$$\hat{w}^*_t = \rho\hat{C}^*_t + \frac{1}{\varphi}\hat{L}^*_t \tag{113}$$

$$\pi^*\hat{\pi}^*_t + w^*\chi\frac{f_X}{A^*}\left(\hat{w}^*_t - \hat{A}^*_t\right) = \frac{1}{\phi}\tilde{Y}^*\frac{\tilde{p}^*(f)}{P^*}\left[\hat{Y}^*_t - \left(\frac{\mu}{\Theta} - 1\right)\hat{\tau}^*_t\right] \tag{114}$$

$$(-\rho)\hat{C}^*_t = -\rho\mathbb{E}_t\left[\hat{C}^*_{t+1}\right] + \mathbb{E}_t\left[\hat{r}^*_{E,t+1}\right] + \left(\frac{2\mu}{\Theta} - 1\right)\left(\hat{\tau}_t - \mathbb{E}_t\left[\hat{\tau}_{t+1}\right]\right) \tag{115}$$

$$\hat{r}^*_{E,t} = (1-\beta)\hat{\pi}^*_t + \beta\hat{v}_t - \hat{v}_{t-1} - \left(\frac{2\mu}{\Theta} - 1\right)\left(\hat{\tau}_{t-1} - \hat{\tau}_t\right) \tag{116}$$

As in Devereux and Sutherland (2007), I will consider $\hat{\xi}_t = \tilde{\alpha}\hat{r}_{x,t}$ as an exogenous *iid* variable. The above equations constitute a system of 22 equations in 22 endogenous variables ($\hat{W}_t$, $\hat{\tau}_t$, $\hat{Y}_t(h)$, $\hat{Y}^*_t(f)$, $\hat{C}_t$, $\hat{C}^*_t$, $\hat{Y}_{D,t}(h)$, $\hat{Y}^*_{D,t}(f)$, $\hat{Y}_{X,t}(h)$, $\hat{Y}^*_{X,t}(f)$, $\hat{w}_t$, $\hat{w}^*_t$, $\hat{L}_t$, $\hat{L}^*_t$, $\hat{l}_t$, $\hat{l}^*_t$, $\hat{\pi}_t$, $\hat{\pi}^*_t$, $\hat{r}_{E,t}$, $\hat{r}^*_{E,t}$, $\hat{z}_{E,t}$, $\hat{z}^*_{E,t}$). There is one endogenous variable predetermined as of time t: the home financial wealth $\hat{W}_t$. As in the benchmark model, there are 3 exogenous variables: the aggregate productivities A_t and A^*_t and the *iid* shock $\hat{\xi}_t$.

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Appendix G

Non-Tradable Goods Model - Closed Form Solution

I follow exactly the same steps as in the benchmark case. Following Devereux and Sutherland (2007), the equation which pins down the solution for optimal portfolios is

$$\mathbb{E}_t\left[\left(\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}\right)\hat{r}_{x,t+1}\right]=0+\mathcal{O}\left(\varepsilon^3\right)$$

It follows that we need to compute $\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}$ and $\hat{r}_{x,t+1}$. Before doing so, some equations need to be modified. First, the total amount of labor in the economy, L_t, will be divided in two parts: the production part l_t (the only part in the benchmark model) and the units of labor paid as fixed costs by the exporting firms, $\frac{N_X f_X}{A_t}$. This division of labor will induce a difference in the dynamics of the terms of trade:

$$\left(1-\frac{\mu}{\Theta}\right)\hat{\tau}_t=\left[1+\frac{1}{\varphi}\left(2\frac{l}{L}-1\right)\right]\hat{A}_t-\rho\hat{C}_t-\frac{1}{\varphi}\hat{Y}_t$$

with $\Theta=\left\{\mu+(1-\mu)\,\chi^{\frac{1-\theta}{1-\phi}}\left(\frac{t}{\eta}\right)^{1-\theta}\right\}$. This Θ is equivalent to Ψ in the benchmark model, except that it takes into account the proportion of exporting firms χ. Note that the only difference with the benchmark model is the presence of the term $\left(2\frac{l}{L}-1\right)$. If all the amount of labor is used for production (there are no fixed export costs), then $l=L$ and this term becomes 1. In this case, we obtain exactly the benchmark model. Following this modification, The first equation which is modified is the differential in output:

$$\begin{aligned}\mathbb{Y}_{t+1} &= \left(\frac{2\mu}{\Theta}-1\right)\mathbb{C}_{t+1}+\frac{4\mu\theta}{\Theta}\left(1-\frac{\mu}{\Theta}\right)\hat{\tau}_{t+1}\\ &= \left(\frac{2\mu}{\Theta}-1\right)\mathbb{C}_{t+1}+\frac{2\mu\theta}{\Theta}\left[\left(1+\frac{1}{\varphi}\left(2\frac{l}{L}-1\right)\right)\mathbb{A}_{t+1}-\rho\mathbb{C}_{t+1}-\frac{1}{\varphi}\mathbb{Y}_{t+1}\right].\end{aligned}$$

Then all the reasoning for the term $\hat{C}_{t+1}-\hat{C}^*_{t+1}-\frac{\hat{Q}_{t+1}}{\rho}$ is similar as before. At the steady state, the total profits in the home country are

$$\pi=\frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}-N_X\frac{wf_X}{A}$$

Intuitively, the first term on the right hand side is the total profit if there are no fixed costs for exporting. Then, these fixed costs are substracted to obtain the net total profits in the home economy. Write equation [101] in a slightly different form

$$\pi\hat{\pi}_t+N_X\frac{wf_X}{A}\left(\hat{w}_t-\hat{A}_t\right)=\frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}\left[\hat{Y}_t+\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t\right]$$

From equation [97] we know that $\left(\hat{w}_t-\hat{A}_t\right)=\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t$. It then follows that

$$\begin{aligned}\pi\hat{\pi}_t+N_X\frac{wf_X}{A}\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t &= \frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}\left[\hat{Y}_t+\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t\right]\\ \pi\hat{\pi}_t &= \frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}\hat{Y}_t+\left[\frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}-N_X\frac{wf_X}{A}\right]\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t\\ \pi\hat{\pi}_t &= \frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}\,(h)}{P}\hat{Y}_t+\pi\left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t\end{aligned}$$

Divide by π

$$\begin{aligned}\hat{\pi}_t &= \frac{\frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}(h)}{P}}{\pi}\hat{Y}_t + \left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t \\ &= \Upsilon\hat{Y}_t + \left(\frac{\mu}{\Theta}-1\right)\hat{\tau}_t,\end{aligned}$$

with $\Upsilon = \frac{\frac{1}{\phi}N_D\tilde{Y}\frac{\tilde{p}(h)}{P}}{\pi}$. This equation is similar to its correspondent in the benchmark case, except one term Υ. We recognize easily that this term is larger or equal to 1. In case that the term is equal to 1 (no fixed export costs), then we will have exactly the same equation as in the benchmark case.

We can use now the closed form solution in Devereux and Sutherland (2007) to obtain

$$\tilde{\alpha} = \frac{(2\mu\theta\rho - 2\mu - \rho\Theta + \Theta)(-\Theta + \mu)}{(-1+\beta)(4\Upsilon\mu^2\theta\rho - 4\Upsilon\mu^2 - 4\mu\rho\Theta\theta\Upsilon + 4\Upsilon\mu\Theta + \rho\Theta^2 - \Upsilon\Theta^2)}$$

Compute then the proportion of home equity held by home residents:

$$\begin{aligned}\alpha_t^E &= \frac{\alpha_{E,t} + N_D v_t}{N_D v_t} = \frac{\tilde{\alpha}\beta N_D Y(h)\frac{p(h)}{P} + N_D\frac{\beta}{1-\beta}\pi}{N_D\frac{\beta}{1-\beta}\pi} = 1 + \frac{\tilde{\alpha}N_D Y(h)\frac{p(h)}{P}}{N_D\frac{1}{1-\beta}\pi} \\ &= 1 + \tilde{\alpha}\phi\Upsilon(1-\beta)\end{aligned}$$

Replace $\tilde{\alpha}$ to obtain

$$\alpha_t^E = 1 - \frac{(2\theta\mu\rho - 2\mu - \rho\Theta + \Theta)(-\Theta + \mu)\Upsilon\phi}{(4\Upsilon\mu^2\theta\rho - 4\Upsilon\mu^2 - 4\mu\rho\Theta\theta\Upsilon + 4\Upsilon\mu\Theta + \rho\Theta^2 - \Upsilon\Theta^2)}$$

Do similar calculus as in the benchmark case

$$\begin{aligned}&1 - \frac{(2\theta\mu\rho - 2\mu - \rho\Theta + \Theta)(-\Theta + \mu)\phi}{\left(4\mu^2\theta\rho - 4\mu^2 - 4\mu\rho\Theta\theta + 4\mu\Theta + \frac{\rho}{\Upsilon}\Theta^2 - \Theta^2\right)} \\ = \; &\frac{1}{2} + \frac{1}{2} - \frac{(2\theta\mu\rho - 2\mu - \rho\Theta + \Theta)(-\Theta + \mu)}{\left(4\mu^2\theta\rho - 4\mu^2 - 4\mu\rho\Theta\theta + 4\mu\Theta + \frac{\rho}{\Upsilon}\Theta^2 - \Theta^2\right)}\phi\end{aligned}$$

Intermediary result:

$$\alpha_t^E = \frac{1}{2} + \frac{\Sigma + 2(\Theta - \mu)(2\theta\mu\rho - 2\mu + \Theta - \rho\Theta)}{2\Sigma} + (\phi - 1)\frac{(\Theta - \mu)(2\theta\mu\rho - 2\mu + \Theta - \rho\Theta)}{\Sigma}$$

with $\Sigma = 4\mu^2\theta\rho - 4\mu^2 - 4\mu\rho\Theta\theta + 4\mu\Theta + \frac{\rho}{\Upsilon}\Theta^2 - \Theta^2$. Note that Σ is similar to Γ obtained in the benchmark case. Take the numerator of the second term:

$$\begin{aligned}\Sigma + 2(\Theta - \mu)(2\theta\mu\rho - 2\mu + \Theta - \rho\Theta) &= \frac{\rho}{\Upsilon}\Theta^2 + \Theta^2 - 2\rho\Theta^2 - 2\mu\Theta + 2\mu\rho\Theta \\ &= \frac{\rho}{\Upsilon}\Theta^2 - \rho\Theta^2 + \Theta^2 - \rho\Theta^2 - 2\mu\Theta + 2\mu\rho\Theta \\ &= \rho\Theta^2\left(\frac{1}{\Upsilon} - 1\right) + (\rho - 1)\Theta(2\mu - \Theta)\end{aligned}$$

Finally, obtain the closed form solution for home equity holdings:

$$\frac{1}{2} + \frac{(\rho-1)}{2}\frac{\Theta(2\mu - \Theta)}{\Sigma} + (\phi - 1)\frac{(\Theta - \mu)(2\theta\mu\rho - 2\mu + \Theta - \rho\Theta)}{\Sigma} + \frac{\rho\left(\frac{1}{\Upsilon} - 1\right)}{2}\frac{\Theta^2}{\Sigma}.$$

Note that in this case the share of capital income in total income is defined as

$$\frac{\pi}{N_D \tilde{Y} \frac{\tilde{p}(h)}{P}} = \frac{1}{\Upsilon \phi}$$

It then follows that the share of labor income Λ is $1 - \frac{1}{\Upsilon \phi}$.