Global Public Signals, Heterogeneous Beliefs and Stock Markets Comovement*

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Abstract

We build an information-based two-country general equilibrium model. There are two dividend processes with correlated growth rates. Agents from each country observe a global public signal informative about both growth rates. We first let agents rationally process information, and then we allow for reasonable departures from rationality. In particular, agents are overconfident with respect to the signal, and as a result have heterogeneous beliefs. In the rational case we report a significant increase in comovement between stock returns. This result is significantly amplified when considering a small amount of misinterpretation of the global signal. Additionally, we find that overconfidence leads to home equity bias. We show that home bias is further increased when fundamentals are more and more correlated. On the technical side, a new methodology for deriving the general equilibrium results is described.

1 Introduction

In recent years, we witnessed a significant increase in the correlation of returns across national stock markets. This raises the natural question of whether the correlations in returns are justified by the (lower) correlations between countries' economic fundamentals. On the one hand, researchers asked themselves whether this increase in correlation could be seen as a result of gradual international financial market integration[1]. On the other hand, this surge in cross-country correlations could also be the result of an increase of the volume of global public information available to investors.

In this paper, we try to isolate the impact of global economic news on the comovement of international financial markets. That is, we theoretically revisit the role of global public news about fundamentals in cross-country correlations. Our results are first derived within a rational setting. We then alter this setup to account for small departures from rationality.

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[1] see, for instance, Dumas, Harvey and Ruiz (2003)

More precisely, irrationality is modeled as a difference in information processing as in Dumas et al. [2009], for instance.

The point of this paper is made in two steps. First, in a benchmark model, perfectly rational agents filter out the unobservable growth rate of dividends (or the fundamental), after observing dividends as well as a global public signal pertaining to the fundamental of each country. The model is solved in closed form and solutions are provided for stock prices, cross-country stock markets comovement and volatilities. The increase in the correlation between the global economic news and the fundamentals can be interpreted as an increase in the relevance of the available global information. The latter increase endogenously produces substantial comovement between stock returns along with an increase in returns volatility. We measure the correlation in different states of the economy, and find that when agents infer that the economy is in a bad state (i.e., they believe that both growth rates are low compared their long term mean), the stock market comovement tends to increase, which tangentially links our paper to the "international contagion" literature.

Second, we consider the case in which both agents perceive their own fundamental process to be differently correlated with the global signal. In particular, each agent overstates the correlation between the global signal and his own fundamental. This generates differences of beliefs across agents. While in this case the computations get more involved, we can still solve the model by keeping the setup affine and applying the associated solution techniques. We derive results in terms of comovement, volatilities and international portfolio choice. We find that the overconfidence of investors amplifies the international stock market comovement as well as the volatility of the stock markets. Moreover, an overstatement of the correlation between the global signal and the domestic fundamental make investors tilt their portfolio towards domestic assets, thus producing home equity bias. For small deviations from rationality, the home equity bias is reinforced if the real correlation between growth rates is high. This result is very intuitive and easy to understand within a standard Markowitz portfolio allocation problem: if two assets become more and more correlated, the diversification benefits are greatly reduced; thus, a small asymmetry in portfolio choice which initially pushes agents to hold a particular asset will be amplified by the increase in correlation. In other words, the correlation between the fundamentals might amplify the home equity bias.

The main contribution of the present work is that we are able to disentangle the effect on comovement of different factors. We show first what fraction is explained by the correlation of the fundamentals, then what percentage comes from the global public news and, finally, we gauge how small deviations from rationality might as well influence comovement. Aditionally, as it will be described in section 3, our solution method is somewhat new and requires the transformation of a system or Riccati equations in a single matrix Riccati equation and, then, to find the closed form in only one step. As it appears that the solution method can be applied to most of general equilibrium models with affine processes, we already exploit it in subsequent work. Finally, we document a mechanism which can be of significant importance in the home equity bias puzzle. Namely, we show that for small deviations from rationality, the correlation between fundamentals may act as an amplifier of the home bias.

Our paper is connected to several strands of literature. The most related research is Dumas et al. [2008], with two main differences. First, we favor global news rather than competitive signals to explain international stock markets comovement. Second, our objective is different, since we want to explain how the global information and its interpretation influences the comovement of international stock markets. Our precise focus on international stock markets comovement requires a richer model than theirs. While their model considers two independent economies à la Dumas et al. [2009], our model bring synergies in their setup by correlating fundamentals. Although this appears to be a straightforward extension, the solution method requires total rethinking, as it will be described in section 3. Other related papers are Karolyi and Stulz [2008] and Veldkamp [2006].

We proceed as follows. In section 2 we build the benchmark model and derive our first results. In section 3 we consider departures from rationality and proceed to describe the solution methodology. Section 4 presents the comovement results. Section 5 presents the home-bias results. Section 6 concludes.

2 Benchmark Model

This section describes a two country model in which we isolate the effect of the global public information on the comovement of asset prices. There is a representative agent located in each of the two countries labeled A and B, respectively. Agents are provided with free access to both financial markets, without any transaction fees. Agents in each country are endowed with one share of their own dividend process. We consider the aggregate endowment process

$$\frac{d\delta_{A,t}}{\delta_{A,t}} = f_{A,t}dt + \sigma_\delta dZ^{\delta}_{A,t}, \tag{1}$$

$$df_{A,t} = \zeta\left(f - f_{A,t}\right)dt + \sigma_f dZ^{f}_{A,t}, \tag{2}$$

for country A, and

$$\frac{d\delta_{B,t}}{\delta_{B,t}} = f_{B,t}dt + \sigma_\delta dZ^{\delta}_{B,t}, \tag{3}$$

$$df_{B,t} = \zeta\left(f - f_{B,t}\right)dt + \sigma_f \rho dZ^{f}_{A,t} + \sigma_f\sqrt{1-\rho^2}dZ^{f}_{B,t}, \tag{4}$$

for country B. Notice that ζ, σ_δ, $\sigma_f > 0$, and that the fundamental processes are correlated. Investors cannot observe the drift of dividends from each country. That is, since the fundamental process is a hidden state, agents of both countries need to filter it out. They do so by using the observable dividend process along with a global public signal s_t. This signal obeys the dynamics

$$ds_t = \rho_H dZ^{f}_{A,t} + \frac{\rho_F - \rho_H\rho}{\sqrt{1-\rho^2}}dZ^{f}_{B,t} + \sqrt{1 - \frac{\rho_H^2 + \rho_F^2 - 2\rho_H\rho_F\rho}{1-\rho^2}}dZ^{s}_{t} \tag{5}$$

The coefficients of the brownians in the signal structure are fixed in such a way that ρ_H will be the correlation of the signal with the home growth rate and ρ_F the correlation of the global signal with the foreign growth rate. In this section only, we shall consider the case in which $\rho_F = \rho_H$. Put differently, the global signal has the same correlation with the processes of the fundamentals $f_{A,t}$ and $f_{B,t}$. This makes the model completely symmetric, and thus provide us with a proper benchmark to gauge the effect of a variation in both ρ_H and ρ_F on the comovement of stock markets of both countries.

The uncertainty in the model is governed by the vector of independent Brownian motions

$$\left[Z_A^\delta,\, Z_B^\delta,\, Z_A^f,\, Z_B^f,\, Z^s\right]'$$

under the objective probability measure. The investor of each country knows that both fundamentals are positively correlated with the global signal s_t, and that the fundamental processes are correlated. They do not under or overestimate any of the correlations stated above. Thus, our benchmark setup is a rational expectation general equilibrium model.

2.1 Information structure and filtering

As stated above, agents in each country have correct beliefs about the information contained in the global signal. Therefore, when performing their filtering, both agents come up with the same estimation for the conditional means of the growth of dividends, $\hat{f}_{A,t}$ and $\hat{f}_{B,t}$. Following Liptser and Shiryaev [2001], the unobservable process is denoted by the vector $d\theta_t^\top = \left[\begin{array}{cc} df_{A,t} & df_{B,t} \end{array}\right]$, and the observable process is collected in the vector $d\xi_t^\top = \left[\begin{array}{ccc} d\delta_{A,t}/\delta_{A,t} & d\delta_{B,t}/\delta_{B,t} & ds_t \end{array}\right]$. The details of the filtering derivations which follow Theorem 12.7 from Liptser and Shiryaev [2001] are provided in Appendix, section 6.1. For simplicity, we only write the conditional expected values, $\hat{f}_{A,t}$ and $\hat{f}_{B,t}$, according to individual of both countries:

$$\left[\begin{array}{c} d\hat{f}_{A,t} \\ d\hat{f}_{B,t} \end{array}\right] = \left(\left[\begin{array}{c} \zeta f \\ \zeta f \end{array}\right] - \left[\begin{array}{cc} \zeta & 0 \\ 0 & \zeta \end{array}\right]\left[\begin{array}{c} \hat{f}_{A,t} \\ \hat{f}_{B,t} \end{array}\right]\right) dt + \left[\begin{array}{ccc} \frac{\gamma}{\sigma_\delta^2} & \frac{\gamma_{HF}}{\sigma_\delta^2} & \rho_H \sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta^2} & \frac{\gamma}{\sigma_\delta^2} & \rho_H \sigma_f \end{array}\right]\left[\begin{array}{c} \frac{d\delta_{A,t}}{\delta_{A,t}} - \hat{f}_{A,t} dt \\ \frac{d\delta_{B,t}}{\delta_{B,t}} - \hat{f}_{B,t} dt \\ ds_t \end{array}\right] \tag{6}$$

where γ is the steady-state variance of $\hat{f}_{A,t}$ and $\hat{f}_{B,t}$, and γ_{HF} is the steady-state covariance of $\hat{f}_{A,t}$ and $\hat{f}_{B,t}$. As in Scheinkman and Xiong [2003], we assume for simplicity that there has been a sufficiently long period of learning for people of both countries to converge to their steady-state level of variance, irrespective of their prior. Thus, these variances and covariances will be constant rather than deterministic functions of time. Note that, since $\rho_H = \rho_F$, the steady-state variances of $\hat{f}_{A,t}$ and $\hat{f}_{B,t}$ are equal. Closed form solutions for γ and γ_{HF} are provided in Appendix, section 6.1.

Since, in the particular case of this section, there is no difference of beliefs, both agents consider the same probability measure. Because agents know the right correlations $\rho_H = \rho_F$ and ρ, the probability measure of both agents matches the objective measure in the economy. Accordingly, we consider the vector $W_t = \left[\begin{array}{ccc} W_{A,t}^\delta & W_{B,t}^\delta & W_t^s \end{array}\right]$ of standard Brownian motions under the probability measure that reflects the physical probability of states of nature. We can then replace the observable process in the filtered drifts for both agents to obtain

$$\begin{bmatrix} d\hat{f}_{A,t} \\ d\hat{f}_{B,t} \end{bmatrix} = \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} - \begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix} \begin{bmatrix} \hat{f}_{A,t} \\ \hat{f}_{B,t} \end{bmatrix} \right) dt + \begin{bmatrix} \frac{\gamma}{\sigma_\delta} & \frac{\gamma_{HF}}{\sigma_\delta} & \rho_H \sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\gamma}{\sigma_\delta} & \rho_H \sigma_f \end{bmatrix} \begin{bmatrix} dW^\delta_{A,t} \\ dW^\delta_{B,t} \\ dW^s_t \end{bmatrix} \tag{7}$$

The two dividend processes are accordingly given by

$$\frac{d\delta_{A,t}}{\delta_{A,t}} = \hat{f}_{A,t}dt + \sigma_\delta dW^\delta_{A,t}, \tag{8}$$

$$\frac{d\delta_{B,t}}{\delta_{B,t}} = \hat{f}_{B,t}dt + \sigma_\delta dW^\delta_{B,t}. \tag{9}$$

The Markovian system made of equations (7) and (8)-(9) completely characterizes the dynamics of the vector of four exogenous state variables which drives the economy. We define this state vector as $X_t = \begin{bmatrix} \delta_{A,t} & \delta_{B,t} & \hat{f}_{A,t} & \hat{f}_{B,t} \end{bmatrix}^\top$. We denote by Σ_t the diffusion matrix of the entire system, and we provide details in Appendix section 6.1.

2.2 Equilibrium

Both agents have power utility with the same risk aversion, $1-\alpha$, and rate of impatience ϕ. Assuming complete financial markets, we may formulate optimization problems using the martingale, "static" approach (as done in Cox and Huang [1989] and Karatzas et al. [1987]). Accordingly, the problem for agent B is

$$\sup_c \mathbb{E} \int_0^\infty e^{-\phi t} \frac{1}{\alpha} \left(c^B_t\right)^\alpha dt; \quad \alpha < 1 \tag{10}$$

$$s.t. \quad \mathbb{E} \int_0^\infty \xi_t c^B_t dt = \mathbb{E} \int_0^\infty \xi_t \delta_{B,t} dt, \tag{11}$$

where ξ_t denotes the state-price density. The optimal consumption policy is given by

$$c^B_t = \left(\lambda_B e^{\phi t} \xi_t\right)^{-\frac{1}{1-\alpha}}, \tag{12}$$

where λ_B denotes the Lagrange multiplier of the static budget constraint. Agent A faces an analogous optimization problem. The first-order condition for its consumption policy is given by

$$c^A_t = \left(\lambda_A e^{\phi t} \xi_t\right)^{-\frac{1}{1-\alpha}}. \tag{13}$$

The equilibrium condition materialized by the aggregate resource constraint is

$$\left(\lambda_B e^{\phi t} \xi_t\right)^{-\frac{1}{1-\alpha}} + \left(\lambda_A e^{\phi t} \xi_t\right)^{-\frac{1}{1-\alpha}} = \delta_{A,t} + \delta_{B,t}. \tag{14}$$

Solving for ξ_t yields

$$\xi_t\left(\delta_{A,t}, \delta_{B,t}\right) = e^{-\phi t} \left[\left(\frac{1}{\lambda_A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} \left(\delta_{A,t} + \delta_{B,t}\right)^{\alpha-1}. \tag{15}$$

Therefore, the consumption shares will be linear in the aggregate dividend. Because we do not consider terms of trade, the aggregate dividend is simply given by the sum of dividends $\delta_{A,t}$ and $\delta_{B,t}$.We have

$$
\begin{aligned}
c_t^A &= \omega\left(\delta_{A,t}+\delta_{B,t}\right), & (16)\\
c_t^B &= (1-\omega)\left(\delta_{A,t}+\delta_{B,t}\right), & (17)
\end{aligned}
$$

where ω is the optimal share of consumption of investor A in aggregate output:

$$
\omega=\frac{\left(\frac{1}{\lambda_A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{1}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}}. \tag{18}
$$

2.3 Securities Market Implementation

Because there are three Brownians that agents care about, we need three securities with linearly independent payoffs along with a riskfree asset to complete the market. As in Dumas et al. [2008], we consider two country stocks which are contingent claims to each country's dividends. In addition to the country stocks, we consider one riskless instantaneous bank deposit with interest rate r_t and a zero net supply futures contract which is marked to the fluctuations of the global signal. Prices of single-payoff stocks, which pay dividends only once in the fixed period T, are functions of $\delta_{A,t},\ \delta_{B,t},\ \widehat{f}_{A,t}$ and $\widehat{f}_{B,t}$:

$$
\begin{aligned}
S_{A,t}^T &= \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\delta_{A,T}\right]=\frac{e^{-\phi(T-t)}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\mathbb{E}_t\left[\delta_{A,T}\left(\delta_{A,T}+\delta_{B,T}\right)^{\alpha-1}\right], & (19)\\
S_{B,t}^T &= \mathbb{E}_t^B\left[\frac{\xi_T}{\xi_t}\delta_{B,T}\right]=\frac{e^{-\phi(T-t)}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\mathbb{E}_t\left[\delta_{B,T}\left(\delta_{A,T}+\delta_{B,T}\right)^{\alpha-1}\right]. & (20)
\end{aligned}
$$

Stock prices are then computed as the sum of the single-payoff stocks, i.e.

$$
\begin{aligned}
S_{A,t}\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t}\right) &= \int_t^\infty S_{A,t}^u\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t}\right)du, & (21)\\
S_{B,t}\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t}\right) &= \int_t^\infty S_{B,t}^u\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t}\right)du. & (22)
\end{aligned}
$$

Single-payoff stock prices are computed by transform analysis. More precisely, consider $\zeta_{A,t}=\ln\delta_{A,t}$ and $\zeta_{B,t}=\ln\delta_{B,t}$. We notice that the setting is affine with respect to the vector

$$
Y_t=\left[\begin{array}{cccc}\zeta_{A,t} & \zeta_{B,t} & \widehat{f}_{A,t} & \widehat{f}_{B,t}\end{array}\right]'.
$$

The general process follows the diffusion dynamics

$$
dY_t=\left(K_0+K_1Y_t\right)dt+\Omega_t dW_t, \tag{23}
$$

with matrices K_0, K_1 and Ω_t provided in the Appendix section 6.2. Consider some coefficient functions $\alpha(\tau) \in \mathbb{R}$ and $\beta(\tau) \in \mathbb{R}^4$, $\beta(\tau) = \begin{bmatrix} \beta_1(\tau) & \beta_2(\tau) & \beta_3(\tau) & \beta_4(\tau) \end{bmatrix}$. In order to derive solutions for the security prices in (19)-(20), we need to compute the moment generating function for the joint distribution of δ_A and δ_B. Since the setup is standard affine with respect to Y, we may directly postulate the functional form of the moment generating function for the joint distribution of δ_A and δ_B (see Duffie [2008]):

$$\mathbb{E}\left[\exp\left(\begin{bmatrix} \varepsilon_A & \varepsilon_B & 0 & 0 \end{bmatrix} Y_t\right)\right] = \mathbb{E}_t\left[e^{\varepsilon_A \zeta_{A,u} + \varepsilon_B \zeta_{B,u}}\right] = \exp\left(\alpha(u-t) + \beta(u-t) Y_t\right). \tag{24}$$

Let us call this function $H\left(\delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}, \widehat{f}_{B,t}\right)$, and let $\tau = u - t$. By standard arguments, as in Duffie [2008], we need to solve a system of Riccati equations of dimension 5. With slight abuse of notation, this system is of the form

$$\beta'(\tau) = K_1^\top \beta(\tau) + \frac{1}{2}\beta(\tau)^\top D_1 \beta(\tau) \tag{25}$$

$$\alpha'(\tau) = K_0^\top \beta(\tau) + \frac{1}{2}\beta(\tau)^\top D_0 \beta(\tau) \tag{26}$$

where K and D refer to the affine drift and covariance matrix of the state vector Y, i.e. $\mu_t = K_0 + K_1 Y_t$ and $\left(\Omega_t \Omega_t^\top\right)_{ij} = D_{0ij} + D_{1ij} Y_t$. Notice that we implicitly use tensor products as $D_1$is a 3-dimensional matrix. In particular, we let $\beta(\tau)^\top D_1 \beta(\tau)$ be the 4-dimensional vector whose k^{th}element is $\sum_{i,j} \beta_i(\tau) D_{1,ijk} \beta_j(\tau)$. The boundary conditions are $\beta_1(0) = \varepsilon_A$, $\beta_2(0) = \varepsilon_B$, $\beta_3(0) = \beta_4(0) = 0$, and $\alpha(0) = 0$. We first derive solutions for $\beta(\tau)$:

$$\beta_1(\tau) = \varepsilon_A \tag{27}$$

$$\beta_2(\tau) = \varepsilon_B \tag{28}$$

$$\beta_3(\tau) = \frac{\varepsilon_A\left(1 - e^{\tau\zeta}\right)}{\zeta} \tag{29}$$

$$\beta_4(\tau) = \frac{\varepsilon_B\left(1 - e^{\tau\zeta}\right)}{\zeta} \tag{30}$$

The solution for $\alpha(\tau)$ is slightly more complicated and provided in the Appendix section 6.2. As in Dumas et al. [2008], we notice that H can be re-expressed as

$$H\left(\zeta_{A,t}, \zeta_{B,t}, \widehat{f}_{A,t}, \widehat{f}_{B,t}\right) = \exp\left(C_{A,t}\varepsilon_A + C_{B,t}\varepsilon_B + \frac{1}{2}C_{A2,t}\varepsilon_A^2 + \frac{1}{2}C_{B2,t}\varepsilon_B^2 + C_{AB,t}\varepsilon_A\varepsilon_B\right), \tag{31}$$

with $C_{A,t}$, $C_{B,t}$, $C_{A2,t}$, $C_{B2,t}$ and $C_{AB,t}$ being functions of $\zeta_{A,t}$, $\zeta_{B,t}$, $\widehat{f}_{A,t}$, $\widehat{f}_{B,t}$ and τ, provided in the Appendix section 6.2. We recognize the Laplace transform of a 2-dimensional Gaussian random variable distributed as

$$\begin{pmatrix} \zeta_{A,t} \\ \zeta_{B,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{bmatrix} C_{A,t} \\ C_{B,t} \end{bmatrix}, \begin{bmatrix} C_{A2,t} & C_{AB,t} \\ C_{AB,t} & C_{B2,t} \end{bmatrix}\right). \tag{32}$$

It follows that the expectations in (19)-(20) can be computed by double numerical integration, as shown below for $S_{A,t}^{T}$:

$$\begin{aligned} S_{A,t}^{T} &= \frac{e^{-\phi(T-t)}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\mathbb{E}_t\left[e^{\zeta_{A,T}}\left(e^{\zeta_{A,t}}+e^{\zeta_{B,t}}\right)^{\alpha-1}\right] \\ &= \frac{e^{-\phi(T-t)}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\zeta_{A,T}}\left(e^{\zeta_{A,t}}+e^{\zeta_{B,t}}\right)^{\alpha-1}\mathcal{N}\left[\cdot,\cdot\right]d\zeta_{A,T}d\zeta_{B,T}, \quad (33) \end{aligned}$$

where $\mathcal{N}\left[\cdot,\cdot\right]$ refers to the bivariate normal distribution in equation (32). The solution for $S_{B,t}^{T}$ is similar.

2.4 Volatilities and Comovement

In this section, we show how the volatility of the stocks depends on the correlation between the global signal and the fundamentals. Moreover, we measure the effect of the correlation between the global signal and the fundamentals on the comovement of stock markets. To do so, we study the diffusion matrix of the stocks.

The diffusion matrix of the stocks represents the sensitivity or "exposure" to the shocks in the fundamentals and the global signal, $dW_{A,t}^{\delta}$, $dW_{B,t}^{\delta}$, and dW_t^{s}, respectively. To that purpose, we need to find the gradient of the price functions and post-multiply by the diffusion matrix of the state variables, Σ_t. The diffusion matrix is:

$$\Delta_t = \begin{bmatrix} \frac{\partial S_{A,t}}{\partial \delta_{A,t}} & \frac{\partial S_{A,t}}{\partial \delta_{B,,t}} & \frac{\partial S_{A,t}}{\partial \widehat{f}_{A,t}} & \frac{\partial S_{A,t}}{\partial \widehat{f}_{B,t}} \\ \frac{\partial S_{B,t}}{\partial \delta_{B,t}} & \frac{\partial S_{B,t}}{\partial \delta_{B,t}} & \frac{\partial S_{B,t}}{\partial \widehat{f}_{A,t}} & \frac{\partial S_{B,t}}{\partial \widehat{f}_{B,t}} \end{bmatrix} \begin{bmatrix} \delta_{A,t}\sigma_\delta & 0 & 0 \\ 0 & \delta_{B,t}\sigma_\delta & 0 \\ \frac{\gamma}{\sigma_\delta} & \frac{\gamma_{HF}}{\sigma_\delta} & \rho_H\sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\gamma}{\sigma_\delta} & \rho_H\sigma_f \end{bmatrix} \quad (34)$$

Solutions for the gradient of the price functions are provided in the Appendix section 6.2. We denote by $\Delta_{A,t}$ the first line of Δ_t and by $\Delta_{B,t}$ its second line. Accordingly, the variance-covariance matrix for stock returns is given by

$$\Sigma_{stocks} = \begin{bmatrix} \Delta_{A,t}/S_{A,t} \\ \Delta_{B,t}/S_{B,t} \end{bmatrix} \begin{bmatrix} \Delta'_{A,t}/S_{A,t} \\ \Delta'_{B,t}/S_{B,t} \end{bmatrix} \quad (35)$$

2.4.1 Choice of parameter values

The parameter values are adapted from Dumas et al. [2009] and Brennan and Xia [2001]. We make sure that the transversality condition is satisfied for both stock prices. The calibration is reported in the table 1 below.

2.4.2 Results

We first study comovement implications of the benchmark case. The results are exposed in Figure 1. First, we notice the direct increase of comovement due to an actual increase in the correlation between the fundamentals. The result is not surprising as we expect that assets with more correlated fundamentals will co-vary more significantly. Second, and more

Parameters and state values	**Symbol**	**Value**
Long-term average growth rate of aggregate endowment	f	0.025
Volatility of expected growth rate of endowment	σ_f	2.5%
Volatility of aggregate endowment	σ_δ	11%
Mean-reversion parameter	ζ	0.3
Agent A's initial share of aggregate endowment	λ_A/λ_B	1
Time preference parameter for both agents	ϕ	0.15
Relative risk-aversion for both agents	$1-\alpha$	7
The level of aggregate dividends	δ_A, δ_B	1
The average belief about the expected rate of growth	$\widehat{f}_{A,t}$, $\widehat{f}_{B,t}$	0.01

Table 1: Parameter values

interestingly, introducing a global signal rationally interpreted by both investors produces a massive increase in the comovement. As an example, from Figure 1, we see that in the case of no actual correlation between fundamentals, i.e. $\rho = 0$, the introduction of a global signal which is only slightly correlated with the dividend growths, i.e $\rho_H = \rho_F = 0.1$, actually reverses the strong and counter-factual correlation between the stock markets from -0.95 to 0.1. Then, the marginal increase in the strength of the signal appears to have a decreasing, yet still positive, impact on comovement. These results come in line with Karolyi and Stulz [2008]. In their study Karolyi and Stulz [2008] discriminate between "competitive" and "global" information shocks, and favor the latter mechanism in explaining the stock return comovement between Japanese and U.S. markets.

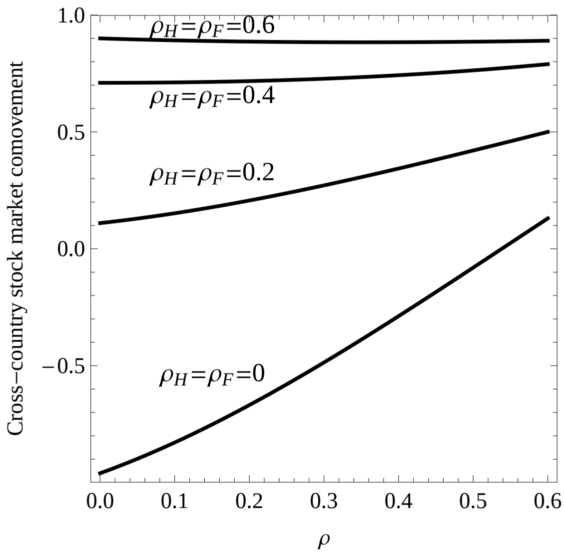

Figure 1: Effect of global news about fundamentals on the comovement between stock returns.

The hypothesis of investors observing and processing the same set of global information is not hard to sustain. Indeed, Veldkamp [2006] shows that, when investors are allowed to endogenously choose the quantity of information they want to buy, they tend to focus on high-demand information that is less expensive. This maximizing behavior clusters the information in a common subset and make stocks move together. The global public signal, although exogenous in our case, could reflect such a concentration of information across investors.

Next we turn to the effect of the inclusion of the global signal on the volatility of stock returns. Note that, since our calibration is symmetric, the volatilities of both stocks are equal. Figure 2 presents the volatility results. As the global signal becomes more informative, i.e. ρ_H and ρ_F simultaneously increase, the exposure of the stocks to the latter signal increases. This translates into an increase of the volatility of returns of the stock prices. This surge is amplified by the price effect: if payoffs are riskier, prices will be smaller, thus the volatility of returns will increase. Increasing informativeness of the global signal can thus be interpreted as an additional source of uncertainty. Put differently, as agents become more aware of additional relevant sources of public information, they take into account more sources of uncertainty. This raises the aggregate amount of uncertainty in the economy. As a result, volatilities systematically increase. The same explanation applies when considering the correlation between the fundamentals, ρ. A high ρ helps investors to better filter the arriving news and thus increases the cross-exposure of stocks to foreign dividend news. This raises the volatility of both stock returns.

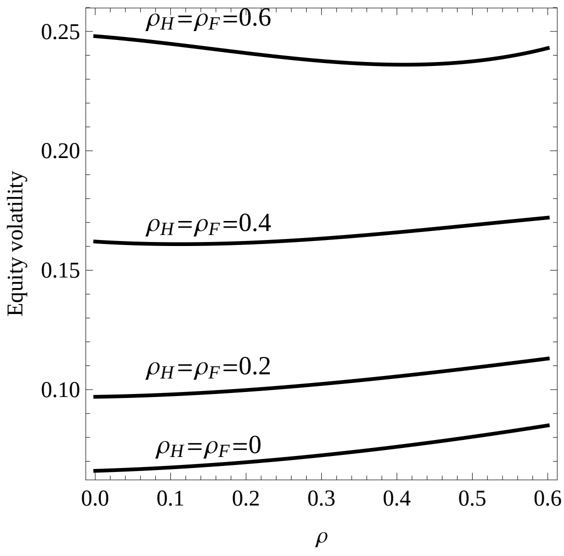

Figure 2: Equity volatility

We are now interested to gauge the impact of the average beliefs about the expected rate of growth of dividends on the correlation between the stock markets. In Figure 3, we compute the cross-country stock market comovement when average beliefs vary from $\hat{f}_A = \hat{f}_B = 0$ to -0.02. We can observe that the impact on comovement due to a decrease in the average beliefs about f_A and f_B is unambiguously positive. When the economy is in a bad state, the stock market comovement tends to increase, a phenomenon commonly referred to as "contagion". We plan to further analyze this issue using Malliavin calculus.

The results so far have been derived under the assumption of rationality. We find first that an increase of the volume of global information endogenously produces significant co-

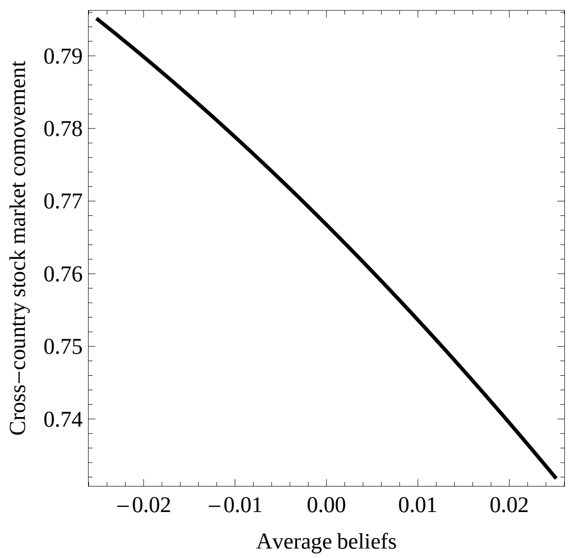

Figure 3: Average beliefs and comovement

movement between stock returns. Second, it increases the amount of volatility in both stock markets. Finally, we observe that a low average belief about the expected rate of growth of dividends generates more comovement. In the next section, we introduce irrational views about the global signal. We emphasize that the departure from rationality that we consider is small in magnitude. Yet, the impact is sizable.

3 Difference of Beliefs

3.1 Information and Beliefs

We now consider the case in which both agents perceive the fundamental process of their own country to be differently correlated with the global signal s. In particular, each agent thinks that the domestic and foreign dividend conditional mean have a correlation of ρ_H and ρ_F with the global signal, respectively. The latter difference of beliefs induces agents to process information differently. More precisely, both agents tend to overstate or understate the correlation of the fundamentals and the global signal with respect to the actual correlation. The latter over or underconfidence brings irrationality into the benchmark setup in which agents are perfectly and rationally inferring the hidden states of the home and foreign economies. Instead, we now let them be subject to bias and allow them to commit mistakes by letting $\rho_H \neq \rho_F$. Hence, the contribution of this section with respect to the benchmark case is to gauge how well irrationality, yet to a reasonable extent, may help to reproduce well-known international finance irregularities. In particular, we expect irrationality to exacerbate comovement among assets. While rational inference about the correlation of both fundamentals was producing some additional comovement, overconfidence with respect to the global signal is likely to magnify this effect. Also, heterogeneity in beliefs introduces an asymmetry across agents and, thus, a bias in portfolio holdings, i.e. our model generates home bias towards domestic assets. This feature was not included in the benchmark model because agents, being fully rational, had perfectly symmetric beliefs. Therefore, perfect diversification obtained and no incentives to deviate from perfect risk-sharing were produced.

Formally, agent A has the following model in mind

$$ds_t = \rho_H dZ^f_{A,t} + \frac{\rho_F - \rho_H \rho}{\sqrt{1-\rho^2}} dZ^f_{B,t} + \sqrt{1 - \frac{\rho_H^2 + \rho_F^2 - 2\rho\rho_H\rho_F}{1-\rho^2}} dZ^s_t \tag{36}$$

whereas, agent B perceives that

$$ds_t = \rho_F dZ^f_{A,t} + \frac{\rho_H - \rho_F \rho}{\sqrt{1-\rho^2}} dZ^f_{B,t} + \sqrt{1 - \frac{\rho_H^2 + \rho_F^2 - 2\rho\rho_H\rho_F}{1-\rho^2}} dZ^s_t. \tag{37}$$

Let $\hat{f}^j_{i,t}$ denote country i's dividend growth rate filtered out by agent j. Since agents perceive the global signal differently, $\hat{f}^j_{i,t}$ is expected to differ across agents. Accordingly, let $\hat{g}_{i,t} = \hat{f}^B_{i,t} - \hat{f}^A_{i,t}$ denote the difference of agent A's beliefs with respect to agent B's beliefs about country i's fundamental. Because we do not wish to postulate that a given agent is irrational, we assume that the objective measure is not defined on either agent's $\sigma-$field. Notice that the objective measure matches the probability measure used in the benchmark case. Again, the assumption of rationality formulated in the previous section implied that agents were processing information according to the physical measure. Although not considering the latter measure, we still need to specify a reference measure which we let to be investor B's. That is, we consider the vector $W^B_t = \begin{bmatrix} W^{\delta,B}_{A,t} & W^{\delta,B}_{B,t} & W^{s,B}_t \end{bmatrix}$ of standard Brownian motions under the probability measure that reflects the expectations of investor B. Then, an application of Kalman-Bucy filtering (Liptser and Shiryaev [2001]) delivers the following filtered dynamics

$$\begin{aligned} \begin{bmatrix} d\hat{f}^B_{A,t} \\ d\hat{f}^B_{B,t} \end{bmatrix} &= \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} - \begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix} \begin{bmatrix} \hat{f}^B_{A,t} \\ \hat{f}^B_{B,t} \end{bmatrix} \right) dt + \\ &+ \begin{bmatrix} \frac{\gamma_F}{\sigma_\delta} & \frac{\gamma_{HF}}{\sigma_\delta} & \rho_F \sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\gamma_H}{\sigma_\delta} & \rho_H \sigma_f \end{bmatrix} \begin{bmatrix} dW^{\delta,B}_{A,t} \\ dW^{\delta,B}_{B,t} \\ dW^{s,B}_t \end{bmatrix} \end{aligned} \tag{38}$$

We leave the derivation of these dynamics to the Appendix, section 6.3. Since dividend growths are unobservable and the objective measure is unknown, the dividend dynamics (1) and (3) may not be directly considered. Rather, we substitute the filtered dividend growth given in (38) into the latter dividend dynamics and write

$$\frac{d\delta_{A,t}}{\delta_{A,t}} = \hat{f}^B_{A,t} dt + \sigma_\delta dW^{\delta,B}_{A,t}, \tag{39}$$

$$\frac{d\delta_{B,t}}{\delta_{B,t}} = \hat{f}^B_{B,t} dt + \sigma_\delta dW^{\delta,B}_{B,t}. \tag{40}$$

Because agent A filters out dividend growths differently, (39) and (40) do not hold under his probability measure. They only do under agent B's measure. Therefore, we need to define a change from the probability measure of investor B to the probability measure of investor A. The differences of beliefs $\hat{g}$ allow to pin down the required change of measure. By Girsanov's theorem, this is achieved through the Radon-Nikodym derivative

$$\begin{aligned} \eta_t &= e^{-\frac{1}{2}\int_0^t \|\nu_u\|^2 du - \int_0^t \nu_u dW_u^B}, && (41) \\ \frac{d\eta_t}{\eta_t} &= -\nu_t dW_t^B, && (42) \end{aligned}$$

with $\nu_t = \frac{1}{\sigma_\delta}\begin{bmatrix} \hat{g}_{A,t} & \hat{g}_{B,t} & 0 \end{bmatrix}^\intercal$.

The law of motion of the difference of beliefs $\hat{g}_{A,t}$ and $\hat{g}_{B,t}$ are obtained using the dynamics filtered by agent A. These are exposed in the Appendix, section 6.3. Putting everything together, the setting is Markovian with respect to the seven state variables: (39), (40), (38), (42) and

$$\begin{aligned} d\hat{g}_{A,t} &= -\left(\left(\zeta + \frac{\gamma_H}{\sigma_\delta^2}\right)\hat{g}_{A,t} + \frac{\gamma_{HF}}{\sigma_\delta^2}\hat{g}_{B,t}\right)dt + \frac{\gamma_F - \gamma_H}{\sigma_\delta} dW_{A,t}^{\delta,B} + (\rho_F - \rho_H)\,\sigma_f dW_t^{s,B} \\ d\hat{g}_{B,t} &= -\left(\frac{\gamma_{HF}}{\sigma_\delta^2}\hat{g}_{A,t} + \left(\zeta + \frac{\gamma_F}{\sigma_\delta^2}\right)\hat{g}_{B,t}\right)dt + \frac{\gamma_H - \gamma_F}{\sigma_\delta} dW_{B,t}^{\delta,B} + (\rho_F - \rho_H)\,\sigma_f dW_t^{s,B} \quad (43) \end{aligned}$$

3.2 Equilibrium

First, notice that agent B's optimization problem is not altered with respect to the problem considered in the benchmark case. However, agent A's problem is modified to account for the change of measure previously defined. That is, agent A's problem remains similar to agent B's problem (10) except that the change of probability measure is now intervening into his objective function. Therefore, agent A's problem becomes

$$\begin{aligned} &\sup_c \mathbb{E}^B \int_0^\infty \eta_t e^{-\phi t}\frac{1}{\alpha}\left(c_t^A\right)^\alpha dt && (44) \\ s.t.\quad &\mathbb{E}^B \int_0^\infty \xi_t^B c_t^A dt = \mathbb{E}^B \int_0^\infty \xi_t^B \delta_{A,t} dt. && (45) \end{aligned}$$

The first-order condition for consumption transforms to

$$c_t^A = \left(\frac{\lambda_A}{\eta_t} e^{\phi t}\xi_t^B\right)^{-\frac{1}{1-\alpha}}. \quad (46)$$

The change of measure η now shows up in agent A's optimal consumption. As usual, we impose the equilibrium condition in the form of the aggregate resource constraint

$$\left(\lambda_B e^{\phi t}\xi_t^B\right)^{-\frac{1}{1-\alpha}} + \left(\frac{\lambda_A}{\eta_t} e^{\phi t}\xi_t^B\right)^{-\frac{1}{1-\alpha}} = \delta_{A,t} + \delta_{B,t}. \quad (47)$$

Solving this equation for the state price density ξ, we obtain:

$$\xi_t^B(\delta_{A,t},\delta_{B,t},\eta_t)=e^{-\phi t}\left[\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}(\delta_{A,t}+\delta_{B,t})^{\alpha-1}. \tag{48}$$

The marginal utility of the representative agent now additionally accommodates for the fact that both agents "agree to disagree" through the change of measure η. Hence, the latter pricing kernel is not only a function of the total output but also reflects the heterogeneity in beliefs. Similar to the benchmark case, we may express individual consumptions as a proportion of total output:

$$c_t^A = \omega(\eta_t)(\delta_{A,t}+\delta_{B,t}), \tag{49}$$

$$c_t^B = [1-\omega(\eta_t)](\delta_{A,t}+\delta_{B,t}), \tag{50}$$

where $\omega(\eta_t)$ is the share of consumption of investor A:

$$\omega(\eta_t)=\frac{\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}}. \tag{51}$$

Notice that, unlike the benchmark case, ω is state-dependent. Indeed, the change of measure induced by the difference of information processing among agents now kicks in into agent A's optimal consumption policy. As a result, the optimal consumption sharing rule exhibits time-variation to account for the changes in difference of beliefs through time.

The menu of assets is not altered with respect to the benchmark case. However, the pricing is different for the reasons mentionned above. The stock prices still satisfy (21)-(22). However, the single-payoff stock prices are given by

$$S_{t,T}^A=e^{-\phi(T-t)}\frac{[1-\omega(\eta_t)]^{1-\alpha}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\sum_{j=0}^{1-\alpha}\Pi_j E_t^B\left[\eta_T^{\frac{j}{1-\alpha}}\delta_{A,T}^{\alpha}\left(1+\frac{\delta_{B,T}}{\delta_{A,T}}\right)^{\alpha-1}\right] \tag{52}$$

for asset A and, similarly,

$$S_{t,T}^B=e^{-\phi(T-t)}\frac{[1-\omega(\eta_t)]^{1-\alpha}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\sum_{j=0}^{1-\alpha}\Pi_j E_t^B\left[\eta_T^{\frac{j}{1-\alpha}}\delta_{B,T}^{\alpha}\left(1+\frac{\delta_{A,T}}{\delta_{B,T}}\right)^{\alpha-1}\right] \tag{53}$$

for asset B, with $\Pi_j=\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega(\eta_t)}{1-\omega(\eta_t)}\right)^j$.

Details of these derivations are provided in the Appendix, section 6.4. The pricing of the stocks requires the computation of the expectation appearing at the very end of the formulae above, an issue to which we turn in the next subsection.

3.3 Transform Analysis

The pricing of the stocks requires the computation of an expectation of the form:

$$\widehat{H}\left(\eta_t, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^B, \widehat{f}_{B,t}^B, \widehat{g}_{A,t}, \widehat{g}_{B,t}, \tau; \varepsilon_A, \varepsilon_B, \chi\right) = \mathbb{E}_t^B\left[(\eta_u)^\chi (\delta_{A,u})^{\varepsilon_A} (\delta_{B,u})^{\varepsilon_B}\right] \tag{54}$$

where $\tau = u - t$. In this subsection, we proceed to describe the derivation of (56). For the sake of clarity, the formal derivation is reproduced in the Appendix, section 6.6.

Unlike Dumas et al. [2009], we do not directly tackle the partial differential equation (PDE) associated with the expectation (54) through Feynman-Kac theorem, i.e. we do not directly solve

$$0 = \mathcal{L}H + \frac{\partial}{\partial t}H$$

where $\mathcal{L}$ stands for the differential generator of $\left(\eta_t, \delta_{A,t}, \delta_{B,t}, \widehat{f}_{A,t}^B, \widehat{f}_{B,t}^B, \widehat{g}_{A,t}, \widehat{g}_{B,t}\right)$. The reason is that we can split, by separation variables, this PDE into a set of simpler ordinary differential equations (ODE). To that purpose, we take advantage of the affine quadratic feature of our model with respect to the state vector

$$Y_t = \left(\zeta_A, \zeta_B, \widehat{f}_A^B, \widehat{f}_B^B, \widehat{g}_A, \widehat{g}_B, \mu\right).$$

Interestingly, we observe that (54) is equivalent to $\mathbb{E}_t^B\left[e^{\varepsilon_A \zeta_{A,u} + \varepsilon_B \zeta_{B,u} + \chi \mu_u}\right]$ when using ζ_A, ζ_B and μ instead of δ_A, δ_B and η. Put differently, the latter expectation takes the form of a Laplace transform for the change of variable introduced in the benchmark section.

Moreover, Cheng and Scaillet [2002] have shown that an affine quadratic setup can be made standard affine by augmenting the state space, i.e. by adding the squares and the cross-product of the g state variables. The diffusion processes of the latter additional state variables, i.e. $\widehat{g}_{A,t}^2$, $\widehat{g}_{B,t}^2$, and $\widehat{g}_{A,t}\widehat{g}_{B,t}$, are obtained by application of Itô's lemma. Accordingly, we prefer to consider the augmented vector process

$$X_t = \left[\zeta_{A,t}, \zeta_{B,t}, \hat{f}_{A,t}^B, \hat{f}_{B,t}^B, \mu_t, \widehat{g}_{A,t}, \widehat{g}_{B,t}, \widehat{g}_{A,t}^2, \widehat{g}_{B,t}^2, \widehat{g}_{A,t}\widehat{g}_{B,t}\right]^\intercal$$

which obeys

$$dX_t = \left(K_0 + K_1 Y X_t\right) dt + \Omega_t \begin{bmatrix} dW_{A,t}^{\delta,B} \\ dW_{B,t}^{\delta,B} \\ dW_t^{s,B} \end{bmatrix} \tag{55}$$

where K_0, K_1 and Ω_t are provided in the Appendix, section 6.5. Since the computation of (54) boils down to the derivation of a Laplace transform within an affine setting, we may follow Duffie [2008] and the references therein and postulate the following moment generating function for the joint distribution of $\zeta_{A,t}$, $\zeta_{B,t}$, and μ_t

$$\mathbb{E}_t^B\left[e^{\varepsilon_A \zeta_{A,u} + \varepsilon_B \zeta_{B,u} + \chi \mu_u}\right] = \exp\left(\alpha\left(u-t\right) + \beta\left(u-t\right) X_t\right). \tag{56}$$

As in the benchmark case, we have introduced some coefficient functions $\alpha(\tau) \in \mathbb{R}$ and

$\beta(\tau) \in \mathbb{R}^{10}$. Denote the latter expectation by

$$H\left(\mu_t, \zeta_{A,t}, \zeta_{B,t}, \widehat{f}_{A,t}^B, \widehat{f}_{B,t}^B, \widehat{g}_{A,t}, \widehat{g}_{B,t}, u-t, \varepsilon_A, \varepsilon_B, \chi\right).$$

Also, similar to the previous section, this function may be obtained by solving a system of Riccati equation which have the same form as (25) and (26). Apparently, this system should be solved much in the same way only taking care of the higher dimensionality. However, not only are there more equations but they also exhibit coupling, i.e. the coefficients functions we are trying to solve for appear in such a way that equations may not be decoupled. This issue is reminiscent of the structure of the diffusion matrix of the state vector Y_t. Indeed, unlike Dumas et al. [2008], we may not partition Y_t into two subsets of exogenous state variables and solve separately for the two economies. That is, (56) may not be viewed as the product of the Laplace transforms associated with each separate economy. Rather, introducing correlation among fundamentals restrict the present system of Riccati equation to be solved globally. Accordingly, our model may not be considered as a general equilibrium with two economies à la Dumas et al. [2009] because of the synergies induced by the correlation between fundamentals.

In that respect, we need to select an appropriate method that allows to solve our problem. First, we observe that $\beta_1 = \varepsilon_A$, $\beta_2 = \varepsilon_B$ and $\beta_5 = \chi$. Those coefficients are directly given by their boundary condition much as in the benchmark derivations. Next, we proceed to transform the system of Riccati equations into a system of matrix Riccati equations and accordingly reorganize the functions of time β_3, β_4, β_6, β_7, β_8, β_9 and β_{10} as elements of the matrix

$$Z = \begin{pmatrix} 0 & \beta_3/2 & 0 & 0 \\ \beta_3/2 & \gamma & \beta_6/2 & \beta_7/2 \\ 0 & \beta_6/2 & \beta_8 & \beta_{10}/2 \\ 0 & \beta_7/2 & \beta_{10}/2 & \beta_9 \end{pmatrix}.$$

This matrix deserves further explanations as it is central to our solving procedure: Rearranging the system of Riccati equations into a system of matrix Riccati equations requires the introduction of an arbitrary coefficient γ which we let to be a function of time. This coefficient, although being completely irrelevant to the computation of (54) and, thus, the equilibrium, serves as a degree of freedom. It is mechanically determined when we impose the system of equations to be a system of matrix Riccati equations. It will prove useful to notice that γ completely identifies every terms containing β_3, β_4, β_6 and β_7 in the ordinary differential equation that α satifies. This is reminiscent of the location of γ in the matrix Z. Also, notice that β_3 appears in the first and second rows of Z and that β_4 has been substituted for $\frac{\varepsilon_B}{\varepsilon_A}\beta_3$. Although, as seen in the benchmark section, β_3 and β_4 are trivially obtained, a closed-form solution to the latter matrix Riccati equation system requires them to be included. Indeed, if we only had considered the system of matrix Riccati equation associated with β_6, β_7, β_8, β_9 and β_{10}, a solution would have been obtained only up to a Magnus series approximation because the coefficient matrix would not have been constant. Rather, our solution is obtained in closed-form. Hence, the inclusion of these coefficient is critical to our solution.

This allows us write our system of Riccati equations as

$$\dot{Z} = J + R^{\top} Z + ZR + ZQZ$$

where the coefficients J, R and Q are provided in the Appendix, section 6.6.

Because we are faced with a system of matrix Riccati equations, we may now apply the usual methodology, for instance as in Fonseca et al. [2008]. More precisely, we invoke Radon's lemma according to which Z is of the form $Z(u-t) = Y(u-t)X(u-t)^{-1}$. This allows to split the system of matrix Riccati equations into a linear Cauchy problem of the form

$$\begin{aligned} \dot{X} &= -RX - QY, \ X(0) = I, \\ \dot{Y} &= JX + R^{\top}Y, \ Y(0) = 0, \end{aligned}$$

where I is the 4×4 identity matrix. The solution to that system is in the form of a matrix exponential:

$$\begin{bmatrix} X(t) & Y(t) \end{bmatrix} = \begin{bmatrix} X(0) & Y(0) \end{bmatrix} \exp\left(\begin{bmatrix} -R & J \\ -Q & R^{\top} \end{bmatrix} t \right).$$

Because this requires the computation of the matrix exponential of a 8×8 matrix, a direct attempt would be tedious. We bypass this issue by considering the Jordan decomposition of the matrix appearing in the matrix exponential. This decomposition allows to rewrite the matrix as $S \exp(J_o t) S^{-1}$ where J_0 denotes the Jordan matrix and S is a so-called similarity matrix pertaining to the Jordan decomposition. This has the major advantage of producing an almost diagonal matrix J_0 and, hence, to considerably facilitate the computation of the matrix exponential.

This is sufficient to pin down each and every coefficients appearing in the Laplace transform, except for the part of the function α that γ does not identify.

The solutions of the β coefficients are given by

$$\begin{aligned} \beta_3(t) &= \frac{\varepsilon_A\left(1-e^{-\zeta t}\right)}{\zeta}, \beta_4(t) = \frac{\varepsilon_B\left(1-e^{-\zeta t}\right)}{\zeta}, \\ \beta_6(t) &= \frac{n_{60} + \sum_{i=1}^{11} n_{6i} e^{b_i t}}{d_0 + \sum_{i=1}^{3} d_i e^{a_i t}}, \beta_7(t) = \frac{n_{70} + \sum_{i=1}^{11} n_{7i} e^{b_i t}}{d_0 + \sum_{i=1}^{3} d_i e^{a_i t}}, \\ \beta_8(t) &= \frac{n_{80} + \sum_{i=1}^{6} n_{8i} e^{c_i t}}{d_0 + \sum_{i=1}^{3} d_i e^{a_i t}}, \beta_9(t) = \frac{n_{90} + \sum_{i=1}^{6} n_{9i} e^{c_i t}}{d_0 + \sum_{i=1}^{3} d_i e^{a_i t}}, \beta_{10}(t) = \frac{n_{100} + \sum_{i=1}^{6} n_{10i} e^{c_i t}}{d_0 + \sum_{i=1}^{3} d_i e^{a_i t}}. \end{aligned}$$

For the sake of tractability, we do not go any further in displaying n, d, a, b and c, not even in the Appendix. Those coefficients are available upon request.

Due to the functional form of the latter coefficients, we are not able to solve for α in closed-form. As mentionned below, only part of α may be analytically derived. Fortunately, a closed-form solution for the remaining part is not necessary for the purpose of calculating the equilibrium and we simply compute it numerically.

Similar to the benchmark case, we notice that H may be re-expressed as

$$H\left(\mu_t, \zeta_{A,t}, \zeta_{B,t}, \widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}, u-t, \varepsilon_A, \varepsilon_B, \chi\right) = \delta_A^{\varepsilon_A}\delta_B^{\varepsilon_B}\eta^{\kappa} \times M_\zeta.$$

M_ζ stands for the Laplace transform of the 2−dimensional Gaussian random variable $(\zeta_{A,t}, \zeta_{B,t})$ distributed as

$$\begin{pmatrix} \zeta_{A,t} \\ \zeta_{B,t} \end{pmatrix} \sim \mathcal{N}\left(\begin{array}{c} \begin{bmatrix} C_A(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) \\ C_B(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) \end{bmatrix} \\ , \begin{bmatrix} C_{A,2}(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) & C_{A,B}(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) \\ C_{A,B}(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) & C_{B,2}(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_{A,t}, \widehat{g}_{B,t}t, u; \chi) \end{bmatrix} \end{array} \right).$$

The C functions are more complicated, though. Moreover, these functions are only obtained in semi closed-form because we solve for α numerically. For these reasons, we do not wish to include them into the discussion.

In order to identify the parts of α pertaining to each C function, we notice that (26) can be rewritten as

$$\alpha'(t) = c_\alpha(t) + c_{\alpha,A}(t)\varepsilon_A + c_{\alpha,B}(t)\varepsilon_B + c_{\alpha,A,2}(t)\varepsilon_A^2 + c_{\alpha,B,2}(t)\varepsilon_B^2 + c_{\alpha,A,B}(t)\varepsilon_A\varepsilon_B.$$

That γ is given in closed-form allows to analytically identify all the coefficients c except for c_α. This should not be a concern for the computation of the equilibrium, though, as c_α may be numerically obtained. On the other hand, although the computation of the equilibrium does not require to dispose of an analytical form for the c coefficient appearing in the 2−dimensional Laplace transform above, their closed-form solution allows to considerably alleviate computations by discarding numerical integration. Again, for the sake of clarity, the c functions are available upon request.

As in the benchmark case, we may rewrite H making explicit use of the joint distribution of $(\zeta_{A,t}, \zeta_{B,t})$. Because we are provided with the joint distribution of ζ_A and ζ_B, any kind of well-behaved expectation may be computed. Hence, this is sufficient to determine the expectations required for the computation of the equilibrium.

4 Comovements

We first proceed to study the comovements among assets in our economy. To that purpose, we need to obtain an expression for the covariance matrix of stocks. The diffusion matrix is obtained much as in the benchmark section except for the gradient of the stock prices to account for a larger dimensionality due to the higher number of state variables. Details are provided in the Appendix section 6.7.

The results obtained in this section are to be compared with the benchmark case $\rho_H = \rho_F = 0.2$. We will proceed as follows. In figure 4 we consider three extents of departure from the rational benchmark case. Those three cases are represented in the figure by dashed lines. First, we observe a significant increase in comovement due to agents' overstating of the correlation between the domestic fundamental and the global signal. We notice that a slight misperception of the latter correlation is sufficient to generate a sizable increase in stock returns comovement. As an example, consider the case in which $\rho = 0$ in figure 4. An increase in ρ_H from 0.2 to 0.4, ceteris paribus, brings the stock markets comovement from 0.1 to 0.5. Moreover, the effect seems to be marginally decreasing. Hence, observed levels of comovement may be consistent with a reasonable degree of misinterpretation of the global public signals.

Also, as highlighted in Dumas et al. [2009], agent B anticipates A's beliefs through ξ^B. Indeed, as previously outlined, the pricing kernel now accommodates for the fluctuations of beliefs. This reflects "higher-order beliefs". To our knowledge, Grisse [2009] is the only paper linking comovement to higher order beliefs. By explicitly modeling higher order beliefs, Grisse [2009] finds that, compared to expectations of fundamentals, expectations of other investors' expectations of fundamentals places more weight on public information. This effect magnifies the spillover of information across assets and generates excess comovement. As previously mentioned, this mechanism is also likely to generate financial contagion as the agents overreact to the global signal about their fundamentals.

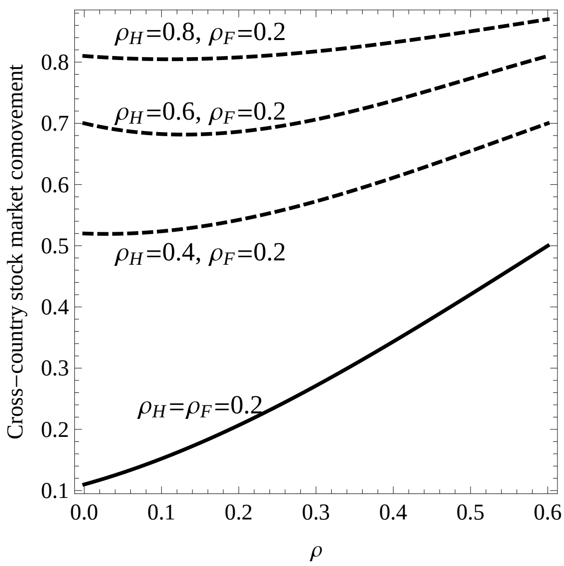

Figure 4: Difference of beliefs and comovement

The effect of difference of beliefs on the stock market volatilities is exposed in figure 5. As overconfidence increases, the equity volatilities increase. The difference of beliefs brings a new risk factor into the model, referred to as "sentiment risk" by Dumas et al. [2009]. The fluctuations in the difference of beliefs increases the trading volume in the market, in turn increasing the volatilities of the stock return.

Both the increase in comovement from figure 4 and the increase in volatility from figure 5 reinforce the fact that one of the causes of the "contagion phenomenon" could be the overconfidence of investors.

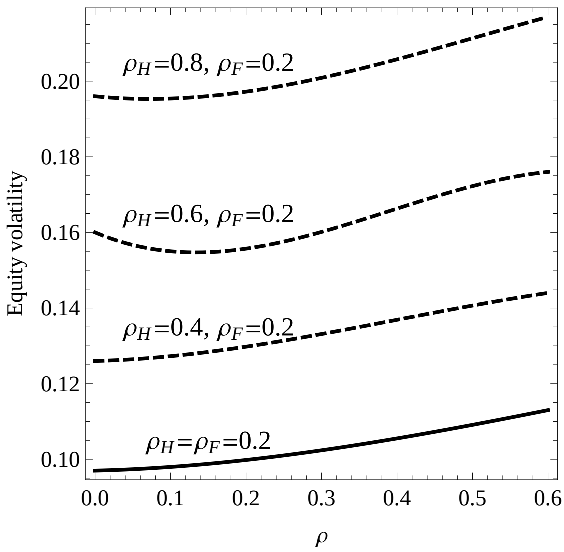

Figure 5: Difference of beliefs and equity volatility

5 Home Bias

We are now interested in checking whether our model is able to generate home bias. Accordingly, we turn to the issue of computing the assets holdings of each population. While the stock prices are computed as in (21)-(22), agent B's financial wealth is given by

$$W_t^B\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t\right)=\int_t^\infty \mathbb{E}_t^B\left[\frac{\xi_u^B}{\xi_t^B}c_{B,u}\right]du. \tag{57}$$

More precisely, we have

$$W_t^B\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t\right)= \\ \frac{(1-\omega(\eta_t))^{1-\alpha}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\sum_{j=0}^{-\alpha}\Upsilon_j H\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t,t,u,\alpha,\frac{j}{1-\alpha},\alpha\right) \tag{58}$$

with $\Upsilon_j=\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega(\eta_t)}{1-\omega(\eta_t)}\right)^j$. The steps are similar to the stock price computation and we postpone them to the Appendix, section 6.7.

We are able to proceed through the computation of the exposure by multiplying the gradient of wealth with respect to the seven state variables with the diffusion of the vector of states. Once the exposures are obtained, we solve a linear system of equations for the portfolio holdings which involves the gradient just derived and the diffusion matrix of stock prices. Again, details are given in the Appendix, section 6.7.

As in Dumas et al. [2008], we construct the measure of home equity bias described in Ahearne et al. [2004]. The results are exposed in figure 6. An overestimation of the correlation between the global signal and the domestic fundamental makes investors tilt their portfolio towards domestic assets. This effect is more pronounced when the overestimation is stronger. Remarkably, for low levels of overconfidence, we observe that the home bias is further increased when fundamentals are more correlated. Considering the economic intuition of portfolio diversification gathered from Markowitz, this result simply states that when international diversification benefits are reduced, investors will have even less incentives to diversify globally. More precisely, from the learning process, overconfidence decreases the perceived variance of the estimated domestic fundamental. This informational advantage will initially make investors tilt their portfolio towards domestic assets. Furthermore, when correlation between fundamentals increases, and thus reducing the benefits of international diversification, the agents will amplify their initial bias and invest even more in domestic assets.

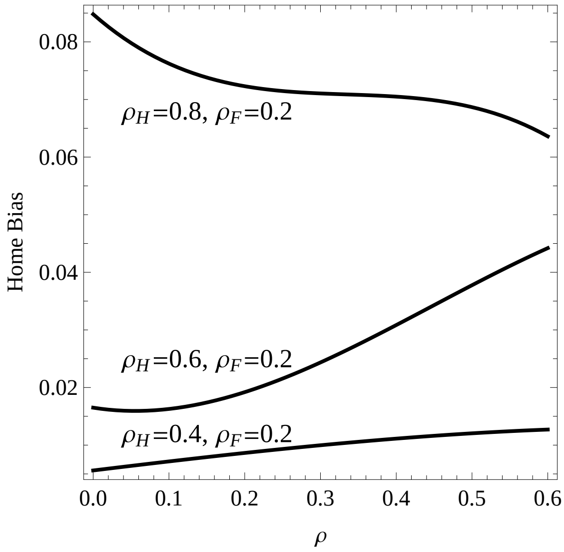

Figure 6: Difference of beliefs and home bias

The fact that we explain equity home bias using overconfidence when Dumas et al. [2008] derive similar results within an underconfidence setting, may be puzzling at first sight. However, both views may be reconciled: in Dumas et al. [2008], the agents are underconfident about the foreign growth rate of dividends, while in our setup agents are overconfident about the domestic growth rate. This makes the two models complementary. Actually, both models can be embedded in a unified setup while maintaining the same number of state variables. This would only increase the dimensionality of the diffusion matrix. In order to be consistent, we performed computations within the latter unified setup and checked that the under and overconfidence effects do not cancel each other out. On the other hand, they do not reinforce each other either. Still, a substantial amount of home bias is achieved within the unified framework, while considering a more reasonable calibration for the Dumas et al. [2008] parameter ϕ.

Figure 7 depicts the home bias for broader ranges of values for ρ_H and ρ_F. More generally, home bias is obtained whenever the absolute value of the perceived correlations, ρ_H is larger than the absolute value of ρ_F. This suggests that home bias is mainly a matter of precision of the estimates of the conditional growth rate of dividends. If investors use a ρ_H larger in absolute value than $|\rho_F|$, their estimate of the domestic growth rate is more precise. Accordingly, they prefer to hold more of the domestic asset. The same reasoning holds if $|\rho_H| < |\rho_F|$, except that foreign equity bias obtains. As figure 7 illustrates, there is a simple relationship between the home or foreign bias and the two perceived correlations, ρ_H and ρ_F. If those two correlations are equal in absolute value, then there is no home or foreign bias. Whenever the absolute values differ, there will be deviations from the optimal market portfolio assuming that the international CAPM holds.

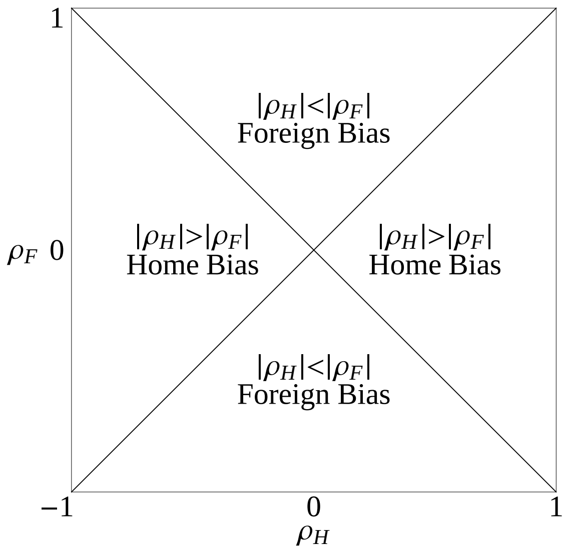

Figure 7: Difference of beliefs and home bias 3

This suggests that home bias generated by heterogeneous beliefs is more a mechanical result. However, the result that correlation between fundamentals might amplify an initial assymetry is very intuitive and reminiscent from the works of Markowitz. Still, to our knowledge, we are the first to document such an intuitive mechanism within a general equilibrium setting in international finance.

6 Conclusion

We presented an information-based model of a two country general equilibrium. As a first result, we show that information, when rationally collected, increases the comovement among asset prices. This effect is due to agents getting more concerned with alternative sources of uncertainty pertaining to both assets. That is, information relevant to both countries is concentrated in a common subset to which agents give more weight. Along with the literature, we interpret this as a consequence of information clustering. Accordingly, we favor a global signal rather than competitive signals in explaining excess stock price comovements. In that respect, our model strongly departs from Dumas et al. [2008]. In particular, using competitive signals, their model does not allow to reproduce excess comovement. This confirms that a global signal appears as a more reasonnable source of international stock markets comovement, as pointed out by Karolyi and Stulz [2008].

Also, we argue that an increase in the flow of information is likely to generate a so-called "contagion" effect. Consistent with Dumas et al. [2009], we show that an increase in the information intensity raises asset prices volatility. Moreoever, as already mentionned and unlike Dumas et al. [2009], we are also able to show that in an economy with multiple assets, such an increase not only produces a surge in volatility but also strengthens the correlation among asset prices. In particular, we demonstrate that bad economic outlooks induce higher levels of comovements. This extends the results derived in Dumas et al. [2009], to an economy with two stocks. In other words, we show that their results not only obtain within a two-tree economy, but that they can be extended to the correlation between assets.

As a second result, we gauge the impact of departures from rationality. More precisely, we let both agents overconfidently process global information with respect to the fundamental of the country in which they are respectively located. We report a significant increase in the asset price comovements due to heterogeneity in beliefs. Remarkably, we observe that a slight deviation from rationality is sufficient to deliver a sizable improvement over the pure rational benchmark. It is critical to observe that underconfidence as modeled in Dumas et al. [2008] would not produce the same effect. On the other hand and consistent with Dumas et al. [2009], time-variation in the difference of beliefs stimulates trades between agents and, therefore, increases asset prices volatility. That is, heterogeneous beliefs magnifies the information impact on volatility and comovement.

We grant evidence that our model is able to produce a substantial amount of home bias. Heterogeneity in beliefs introduces an asymmetry which tilts portfolio holdings towards domestic assets. In particular, we emphasize that home bias is due to a lack of precision in the estimate of each country's fundamental. This provides domestic investors with an informational advantage which gets larger as the correlation between fundamentals increases. Therefore, investors bias their portfolio towards domestic assets as they perceive the benefits of international diversification to be less attractive. On the one hand, we argue that overconfidence and underconfidence in processing information would only prove complementary in a unified model. On the other hand, underconfidence as modeled in Dumas et al. [2008] does not capture the mentionned magnifying effect as fundamentals remain uncorrelated and, hence, keep international diversification as an attractive outlook.

Finally, we technically contribute to the existing literature in designing a general methodology allowing to solve for general equilibrium with multiple stocks and fundamental correlation. Dumas et al. [2009] have shown, within a complete financial markets economy with a single stock and learning, that a general equilibrium could be solved by deriving the Fourier transform of the stochastic processes that make the equilibrium Markovian. Dumas et al. [2008] build on this result to show that this procedure may be extended to the multiple Lucas trees case, yet only when the fundamentals remain uncorrelated. We show how the general case in which fundamentals are correlated may be solved. To that purpose, unlike both articles which attempt to solve the partial differential equation associated with transform in a direct manner, we first extend the state-space to make it affine quadratic much as in Cheng and Scaillet [2002]. We, then, obtain a system of Riccati equations which we recast into a system of matrix Riccati equations. We show that it is feasible to do so provided that one decomposes the system in a very particular way which we carefully describe. This allows to obtain the Fourier transform in closed-form up to an integral which may be computed numerically. Given the importance of the correlation between fundamentals illustrated above, we hope that this approach will prove useful for other applications.

Appendix

6.1 Benchmark Case: Filtering

[TO BE ADDED]

6.2 Benchmark Case: Transform Analysis

[TO BE ADDED]

6.3 Difference of Beliefs: Filtering

[TO BE ADDED]

6.3.1 Agent A

We first wish to determine the filtered dividend growth dynamics associated with agent A's perception. Following Liptser and Shiryaev (2001), we denote the unobservable process by θ_t and write its dynamics as follows:

$$\begin{aligned} d\theta_t = \begin{bmatrix} df_t^A \\ df_t^B \end{bmatrix} &= \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} + \begin{bmatrix} -\zeta & 0 \\ 0 & -\zeta \end{bmatrix} \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + \\ &+ \begin{bmatrix} \sigma_f & 0 \\ \sigma_f \rho & \sigma_f \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix} \\ &= \left(a_0 + a_1 \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + b_1 \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + b_2 \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix}. \quad (59) \end{aligned}$$

The observable process is denoted by ξ_t and obeys the following dynamics:

$$d\xi_t = \begin{bmatrix} \frac{d\delta_t^A}{\delta_t^A} \\ \frac{d\delta_t^B}{\delta_t^B} \\ ds_t \end{bmatrix} = \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \rho_H & \rho_F \end{bmatrix} \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} +$$

$$+ \begin{bmatrix} \sigma_\delta & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sqrt{1-\rho_H^2-\rho_F^2} \end{bmatrix} \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix}$$

$$= \left(A_0 + A_1 \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + B_1 \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + B_2 \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix}. \quad (60)$$

Still using Liptser and Shiryaev (2001) notations, we get

$$b \circ b = b_1 b_1' + b_2 b_2' = \begin{bmatrix} \sigma_f^2 & \rho\sigma_f^2 \\ \rho\sigma_f^2 & \sigma_f^2 \end{bmatrix} \quad (61)$$

$$B \circ B = B_1 B_1' + B_2 B_2' = \begin{bmatrix} \sigma_\delta^2 & 0 & 0 \\ 0 & \sigma_\delta^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (62)$$

$$b \circ B = b_1 B_1' + b_2 B_2' = \begin{bmatrix} 0 & 0 & \rho_H \sigma_f \\ 0 & 0 & \left(\sqrt{1-\rho^2}\rho_F + \rho\rho_H\right)\sigma_f \end{bmatrix}. \quad (63)$$

Applying theorem 12.7 (Liptser and Shiryaev (2001)), we get the following dynamics for estimates

$$\begin{bmatrix} d\hat{f}_t^{A,A} \\ d\hat{f}_t^{B,A} \end{bmatrix} =$$

$$\left(a_0 + a_1 \begin{bmatrix} \hat{f}_t^{A,A} \\ \hat{f}_t^{B,A} \end{bmatrix} \right) dt + \left[(b \circ B) + \gamma_{B,t} A_1' \right] (B \circ B)^{-1} \left[d\xi_t - \left(A_0 + A_1 \begin{bmatrix} \hat{f}_t^{A,A} \\ \hat{f}_t^{B,A} \end{bmatrix} \right) dt \right] \quad (64)$$

$$\dot{\gamma}_t = a_1 \gamma_{A,t} + \gamma_{A,t} a_1' + (b \circ b) - \left[(b \circ B) + \gamma_{A,t} A_1' \right] (B \circ B)^{-1} \left[(b \circ B) + \gamma_{A,t} A_1' \right]' \quad (65)$$

along with covariance matrix

$$\gamma_{A,t} = \begin{bmatrix} \gamma_{H,t} & \gamma_{HF,t} \\ \gamma_{HF,t} & \gamma_{F,t} \end{bmatrix} \quad (66)$$

6.3.2 Variances

As in Scheinkman & Xiong (2003), we assume that agents have gained mature learning such that variances have reached their steady-state level. In particular, agents' posterior variance at the steady state solves the equation

$$0 \quad = \quad a_1\gamma_A + \gamma_A a_1' + (b \circ b) - [(b \circ B) + \gamma_A A_1'] \, (B \circ B)^{-1} \, [(b \circ B) + \gamma_A A_1']' \, . \qquad (67)$$

Denote by γ_H the steady state variance of f^A estimated by the investor A as it pertains to the variance of the estimated drift of the *home* dividends. We have then:

$$\gamma_H = \frac{\sqrt{(\Omega_1+\Omega_2)\sigma_\delta^2 \left(\begin{array}{c} \frac{\left(\rho^2 {\rho_i}^2 + {\rho_j}^2 - 1\right)(\Omega_2-\Omega_1)\sigma_f^2}{2\rho\rho_F\rho_H+1} + \\ \left(\zeta^2(\Omega_1+\Omega_2) - \left(-\rho_F\rho_H\rho^2 + \left({\rho_F}^2 + {\rho_H}^2 - 1\right)\rho + \rho_F\rho_H\right)^2 \sigma_f^4\right)\sigma_\delta^2 \end{array} \right)}}{\Omega 1 - \Omega 3} - \zeta\sigma_\delta^2$$

where $i = F$ and $j = H$ and

$$\Omega_1 \quad = \quad 2\left(\begin{array}{c} (2\rho\rho_F\rho_H\zeta + \zeta)^2\sigma_\delta^4 + \\ \sqrt{(2\rho\rho_F\rho_H + 1)^3\sigma_\delta^4\Omega_4} \end{array} \right),$$

$$\Omega_2 = (-2\rho\rho_F\rho_H - 1)\left(\left(\rho^2+1\right){\rho_F}^2 + \left(\rho^2+1\right){\rho_H}^2 - 2\right)\sigma_f^2\sigma_\delta^2,$$

$$\Omega_3 = \left(2\rho\left(\rho^2+1\right)\rho_H{\rho_F}^3 + \left(\rho^2+1\right){\rho_F}^2 + 2\rho\rho_H\left(\left(\rho^2+1\right){\rho_H}^2 - 2\right)\rho_F + \left(\rho^2+1\right){\rho_H}^2 - 2\right)\sigma_f^2\sigma_\delta^2,$$

and

$$\Omega_4 = \left(\rho^2-1\right)\left({\rho_F}^2 + {\rho_H}^2 - 1\right)\sigma_f^4 - \zeta^2\left(\left(\rho^2+1\right){\rho_F}^2 + \left(\rho^2+1\right){\rho_H}^2 - 2\right)\sigma_\delta^2\sigma_f^2 + \zeta^4(2\rho\rho_F\rho_H+1)\sigma_\delta^4.$$

Similarly, the steady state variance of f^B estimated by the investor A is denoted by γ_F and is obtained just by setting $i = H$ and $j = F$.

Notice that the variances γ_H and γ_F are equal only if $\rho_H^2 = \rho_F^2$. Also, the steady state variance of the estimated drift for the foreign dividends will be larger if $\rho_H^2 > \rho_F^2$.

Finally, the steady state covariance for the estimated f^A and f^B is:

$$\gamma_{HF} = \frac{\left(\rho_F\rho_H\rho^2 - \left({\rho_F}^2 + {\rho_H}^2 - 1\right)\rho - \rho_F\rho_H\right)\sigma_f^2\sigma_\delta^2}{(2\rho\rho_F\rho_H + 1)\sqrt{\frac{\Omega 1 + \Omega 2}{(2\rho\rho_F\rho_H+1)^2}}} \qquad (68)$$

We observe that the covariance will be negative when ρ_H and ρ_F have the same sign and positive otherwise. It will be nihil whenever ρ_H or ρ_F is null.

The filtered drifts for agent A are

$$\begin{bmatrix} d\hat{f}_t^{A,A} \\ d\hat{f}_t^{B,A} \end{bmatrix} = \begin{array}{c} \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} - \begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix} \begin{bmatrix} \hat{f}_t^{A,A} \\ \hat{f}_t^{B,A} \end{bmatrix} \right) dt \\ + \begin{bmatrix} \frac{\gamma_H}{\sigma_\delta^2} & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \rho_H\sigma_f \\ \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \frac{\gamma_F}{\sigma_\delta^2} & \left(\sqrt{1-\rho^2}\rho_F + \rho\rho_H\right)\sigma_f \end{bmatrix} \left(d\xi_t - \left(A_0 + A_1 \begin{bmatrix} \hat{f}_t^{A,A} \\ \hat{f}_t^{B,A} \end{bmatrix} \right) dt \right) \end{array} \qquad (69)$$

6.3.3 Agent B

Proceeding similarly, the unobservable process for agent B is given by

$$d\theta_t = \begin{bmatrix} df_t^A \\ df_t^B \end{bmatrix} = \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} + \begin{bmatrix} -\zeta & 0 \\ 0 & -\zeta \end{bmatrix} \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt +$$

$$+ \begin{bmatrix} \sigma_f & 0 \\ \sigma_f \rho & \sigma_f \sqrt{1-\rho^2} \end{bmatrix} \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix}$$

$$= \left(a_0 + a_1 \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + b_1 \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + b_2 \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix} \quad (70)$$

The observable process is

$$d\xi_t = \begin{bmatrix} \frac{d\delta_t^A}{\delta_t^A} \\ \frac{d\delta_t^B}{\delta_t^B} \\ ds_t \end{bmatrix} = \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \rho_F & \rho_H \end{bmatrix} \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} +$$

$$+ \begin{bmatrix} \sigma_\delta & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ 0 & 0 & \sqrt{1-\rho_H^2-\rho_F^2} \end{bmatrix} \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix}$$

$$= \left(A_0 + A_1 \begin{bmatrix} f_t^A \\ f_t^B \end{bmatrix} \right) dt + B_1 \begin{bmatrix} dZ_{A,t}^f \\ dZ_{B,t}^f \end{bmatrix} + B_2 \begin{bmatrix} dZ_{A,t}^\delta \\ dZ_{B,t}^\delta \\ dZ_t^S \end{bmatrix} \quad (71)$$

Note that the only modified matrix in the case of agent B is B_1. Moreover,

$$b \circ b = b_1 b_1' + b_2 b_2' = \begin{bmatrix} \sigma_f^2 & \rho\sigma_f^2 \\ \rho\sigma_f^2 & \sigma_f^2 \end{bmatrix} \quad (72)$$

$$B \circ B = B_1 B_1' + B_2 B_2' = \begin{bmatrix} \sigma_\delta^2 & 0 & 0 \\ 0 & \sigma_\delta^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (73)$$

$$b \circ B = b_1 B_1' + b_2 B_2' = \begin{bmatrix} 0 & 0 & \rho_F \sigma_f \\ 0 & 0 & \left(\sqrt{1-\rho^2}\rho_H + \rho\rho_F \right) \sigma_f \end{bmatrix} \quad (74)$$

Again, applying theorem 12.7 (Liptser and Shiryaev (2001)), we find:

$$\begin{bmatrix} d\hat{f}_t^{A,B} \\ d\hat{f}_t^{B,B} \end{bmatrix} =$$

$$\left(a_0 + a_1 \begin{bmatrix} \hat{f}_t^{A,B} \\ \hat{f}_t^{B,B} \end{bmatrix}\right) dt + \left[(b \circ B) + \gamma_{B,t} A_1'\right] (B \circ B)^{-1} \left[d\xi_t - \left(A_0 + A_1 \begin{bmatrix} \hat{f}_t^{A,B} \\ \hat{f}_t^{B,B} \end{bmatrix}\right) dt\right] \tag{75}$$

$$\dot{\gamma}_t = a_1 \gamma_{B,t} + \gamma_{B,t} a_1' + (b \circ b) - \left[(b \circ B) + \gamma_{B,t} A_1'\right] (B \circ B)^{-1} \left[(b \circ B) + \gamma_{B,t} A_1'\right]' \tag{76}$$

along with covariance matrix

$$\gamma_{B,t} = \begin{bmatrix} \gamma_{F,t} & \gamma_{HF,t} \\ \gamma_{HF,t} & \gamma_{H,t} \end{bmatrix} \tag{77}$$

Intuitively, we also find the same variances and covariances, except that γ_H represents now the steady state variance of f^B estimated by the investor B, and γ_F represents the steady state variance of f^A estimated by the investor B. The filtered drifts for agent B are as follows:

$$\begin{bmatrix} d\hat{f}_t^{A,B} \\ d\hat{f}_t^{B,B} \end{bmatrix} = \left(\begin{bmatrix} \zeta f \\ \zeta f \end{bmatrix} - \begin{bmatrix} \zeta & 0 \\ 0 & \zeta \end{bmatrix} \begin{bmatrix} \hat{f}_t^{A,B} \\ \hat{f}_t^{B,B} \end{bmatrix}\right) dt +$$
$$+ \begin{bmatrix} \frac{\gamma_F}{\sigma_\delta^2} & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \rho_F \sigma_f \\ \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \frac{\gamma_H}{\sigma_\delta^2} & \left(\sqrt{1-\rho^2}\rho_H + \rho\rho_F\right)\sigma_f \end{bmatrix} \left(d\xi_t - \left(A_0 + A_1 \begin{bmatrix} \hat{f}_t^{A,B} \\ \hat{f}_t^{B,B} \end{bmatrix}\right) dt\right). \tag{78}$$

6.4 Difference of Beliefs: Stock Prices

The stock prices still satisfy (). However, the single-payoff stocks are given by

$$S_{t,T}^A = E_t^B \left[\frac{\xi_T^B}{\xi_t^B} \delta_{A,T}\right] \tag{79}$$

$$= E_t^B \left[\frac{e^{-\delta T}\left[\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} (\delta_{A,T} + \delta_{B,T})^{\alpha-1}}{e^{-\delta t}\left[\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} (\delta_{A,t} + \delta_{B,t})^{\alpha-1}} \delta_{A,T}\right] \tag{80}$$

Using (51), we may write

$$\left(\frac{\eta_t \lambda_B}{\lambda_A}\right)^{\frac{1}{1-\alpha}} = \frac{\omega(\eta_t)}{1-\omega(\eta_t)} \tag{81}$$

and

$$\left[\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} = \left(\frac{1}{1-\omega(\eta_t)}\right)^{1-\alpha} \frac{1}{\lambda_B} \tag{82}$$

Substituting the latter expression into the denominator of (80), we obtain

$$S_{t,T}^{A} = e^{-\delta(T-t)}\frac{\left[1-\omega\left(\eta_t\right)\right]^{1-\alpha}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}E_t^B\left[\frac{\left[\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}\left(\delta_{A,T}+\delta_{B,T}\right)^{\alpha-1}}{\frac{1}{\lambda_B}}\delta_{A,T}\right] \tag{83}$$

$$= e^{-\delta(T-t)}\frac{\left[1-\omega\left(\eta_t\right)\right]^{1-\alpha}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}E_t^B\left[\lambda_B\left[\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}\delta_{A,T}^{\alpha}\left(1+\frac{\delta_{B,T}}{\delta_{A,T}}\right)^{\alpha-1}\right] \tag{84}$$

On the other hand, we know that

$$\left[\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha} = \frac{1}{\lambda_B}\sum_{j=0}^{1-\alpha}\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{\eta_T\lambda_B}{\lambda_A}\right)^{\frac{j}{1-\alpha}} \tag{85}$$

$$= \frac{1}{\lambda_B}\sum_{j=0}^{1-\alpha}\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\eta_t\lambda_B}{\lambda_A}\right)^{\frac{j}{1-\alpha}}\eta_T^{\frac{j}{1-\alpha}} \tag{86}$$

$$= \frac{1}{\lambda_B}\sum_{j=0}^{1-\alpha}\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega\left(\eta_t\right)}{1-\omega\left(\eta_t\right)}\right)^{j}\eta_T^{\frac{j}{1-\alpha}} \tag{87}$$

Inserting this expression into (84), we finally get

$$S_{t,T}^{A} = e^{-\delta(T-t)}\frac{\left[1-\omega\left(\eta_t\right)\right]^{1-\alpha}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\times$$

$$\times E_t^B\left[\sum_{j=0}^{1-\alpha}\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega\left(\eta_t\right)}{1-\omega\left(\eta_t\right)}\right)^{j}\eta_T^{\frac{j}{1-\alpha}}\delta_{A,T}^{\alpha}\left(1+\frac{\delta_{B,T}}{\delta_{A,T}}\right)^{\alpha-1}\right]$$

$$= \begin{array}{c} e^{-\delta(T-t)}\frac{\left[1-\omega(\eta_t)\right]^{1-\alpha}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\sum_{j=0}^{1-\alpha}\begin{pmatrix}1-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}} \\ \times\left(\frac{\omega(\eta_t)}{1-\omega(\eta_t)}\right)^{j}E_t^B\left[\eta_T^{\frac{j}{1-\alpha}}\delta_{A,T}^{\alpha}\left(1+\frac{\delta_{B,T}}{\delta_{A,T}}\right)^{\alpha-1}\right]\end{array} \tag{88}$$

6.4.1 Stock Price B

We have

$$S_{t,T}^{B} = E_t^B\left[\frac{\xi_T^B}{\xi_t^B}\delta_{B,T}\right] \tag{89}$$

$$= E_t^B\left[\frac{e^{-\delta T}\left[\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}\left(\delta_{A,T}+\delta_{B,T}\right)^{\alpha-1}}{e^{-\delta t}\left[\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right]^{1-\alpha}\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\delta_{B,T}\right] \tag{90}$$

Thus, by the same reasoning, we get

$$S_{t,T}^{B} = \begin{array}{c} e^{-\delta(T-t)} \frac{[1-\omega(\eta_t)]^{1-\alpha}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}} \sum_{j=0}^{1-\alpha} \begin{pmatrix} 1-\alpha \\ j \end{pmatrix} \left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}} \\ \times \left(\frac{\omega(\eta_t)}{1-\omega(\eta_t)}\right)^{j} E_t^B \left[\eta_T^{\frac{j}{1-\alpha}} \delta_{B,T}^{\alpha} \left(1+\frac{\delta_{A,T}}{\delta_{B,T}}\right)^{\alpha-1}\right] \end{array} \tag{91}$$

One notices that the only changing element is the function of $\delta_{A,T}$ and $\delta_{B,T}$ located at the end of the expression for the stock prices.

6.4.2 Agent B's wealth

From the derivation of the equilibrium, we substitute the expression for consumption of agent B and obtain

$$\begin{aligned} W_t^B\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t\right) &= \int_t^\infty \mathbb{E}_t^B\left[\frac{\xi_u^B}{\xi_t^B}\left(1-\omega\left(\eta_u\right)\right)\left(\delta_{A,u}+\delta_{B,u}\right)\right]du \qquad (92) \\ &= \int_t^\infty \mathbb{E}_t^B\left[\begin{array}{c} \frac{\left(\left(\frac{\eta_u}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}}{\left(\left(\frac{\eta_t}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}(\delta_{A,t}+\delta_{B,t})^{\alpha-1}} \\ \times\left(1-\omega\left(\eta_u\right)\right)\left(\delta_{A,u}+\delta_{B,u}\right)^{\alpha} \end{array}\right]du. \end{aligned}$$

On the other hand,

$$\begin{aligned} \left(\left(\frac{\eta_T}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}}\right)^{-\alpha}\left(\frac{1}{\lambda_B}\right)^{\frac{1}{1-\alpha}} &= \left(\left(\frac{\eta_T\lambda_B}{\lambda_A}\right)^{\frac{1}{1-\alpha}}+1\right)^{-\alpha}\frac{1}{\lambda_B} \\ &= \frac{1}{\lambda_B}\sum_{j=0}^{-\alpha}\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{\eta_T\lambda_B}{\lambda_A}\right)^{\frac{j}{1-\alpha}} \\ &= \frac{1}{\lambda_B}\sum_{j=0}^{-\alpha}\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\eta_t\lambda_B}{\lambda_A}\right)^{\frac{j}{1-\alpha}}\eta_T^{\frac{j}{1-\alpha}} \\ &= \frac{1}{\lambda_B}\sum_{j=0}^{-\alpha}\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega\left(\eta_t\right)}{1-\omega\left(\eta_t\right)}\right)^{j}\eta_T^{\frac{j}{1-\alpha}} \end{aligned}$$

where the second equality follows from the binomial theorem. We can then substitute this expression into (92) and obtain for

$$\begin{aligned} & W_t^B\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t\right) \\ = & \frac{\left(1-\omega\left(\eta_t\right)\right)^{1-\alpha}}{\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}}\sum_{j=0}^{-\alpha}\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega\left(\eta_t\right)}{1-\omega\left(\eta_t\right)}\right)^{j}\mathbb{E}_t\left[\eta_T^{\frac{j}{1-\alpha}}\delta_{A,T}^{\alpha}\left(1+\frac{\delta_{B,T}}{\delta_{A,T}}\right)^{\alpha}\right]. \end{aligned}$$

Finally, notice that the expectation appearing in the wealth may be computed using the Fourier transform previously derived such that

$$
\begin{aligned}
& W_t^B\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t\right) \\
= \; & \frac{(1-\omega(\eta_t))^{1-\alpha}}{(\delta_{A,t}+\delta_{B,t})^{\alpha-1}}\sum_{j=0}^{-\alpha}\begin{pmatrix}-\alpha\\ j\end{pmatrix}\left(\frac{1}{\eta_t}\right)^{\frac{j}{1-\alpha}}\left(\frac{\omega(\eta_t)}{1-\omega(\eta_t)}\right)^j H\left(\delta_{A,t},\delta_{B,t},\widehat{f}_{A,t},\widehat{f}_{B,t},\eta_t,t,u,\alpha,\frac{j}{1-\alpha},\alpha\right).
\end{aligned}
\tag{93}
$$

6.5 Difference of Beliefs: State Vector

Here, we only provide the details of the matrix K_0, K_1 and Ω_t characterizing the state vector. These are given by:

$$
K_0=\begin{bmatrix}
-\frac{\sigma_\delta^2}{2}\\
-\frac{\sigma_\delta^2}{2}\\
\zeta f\\
\zeta f\\
0\\
0\\
0\\
\frac{(\gamma_F-\gamma_H)^2}{\sigma_\delta^2}+\frac{(\rho-1)^2(\rho_F-\rho_F)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2}\\
\frac{(\gamma_F-\gamma_H)^2}{\sigma_\delta^2}+\frac{(\rho-1)^2(\rho_F-\rho_F)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2}\\
\frac{(\rho-1)^2(\rho_F-\rho_H)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2}
\end{bmatrix},
\tag{94}
$$

$$
K_1=-\begin{bmatrix}
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \zeta & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \zeta & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2\sigma_\delta^2} & \frac{1}{2\sigma_\delta^2} & 0\\
0 & 0 & 0 & 0 & 0 & \zeta+\frac{\gamma_H}{\sigma_\delta^2} & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \zeta+\frac{\gamma_F}{\sigma_\delta^2} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\left(\zeta+\frac{\gamma_H}{\sigma_\delta^2}\right) & 0 & \frac{2\gamma_{\mathrm{HF}}}{\sigma_\delta^2}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\left(\zeta+\frac{\gamma_F}{\sigma_\delta^2}\right) & \frac{2\gamma_{\mathrm{HF}}}{\sigma_\delta^2}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta^2} & 2\zeta+\frac{\gamma_F+\gamma_H}{\sigma_\delta^2}
\end{bmatrix},
\tag{95}
$$

and

$$\Omega_t = \begin{bmatrix} \sigma_\delta & 0 & 0 \\ 0 & \sigma_\delta & 0 \\ \frac{\gamma_F}{\sigma_\delta} & \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta} & \frac{(\rho_F+\rho_H\rho)\sigma_f}{1+2\rho\rho_F\rho_H} \\ \frac{\gamma_{\mathrm{HF}}}{\sigma_\delta} & \frac{\gamma_H}{\sigma_\delta} & \frac{(\rho_H+\rho_F\rho)\sigma_f}{1+2\rho\rho_F\rho_H} \\ -\frac{\widehat{g}_{A,t}}{\sigma_\delta} & -\frac{\widehat{g}_{B,t}}{\sigma_\delta} & 0 \\ \frac{\gamma_F-\gamma_H}{\sigma_\delta} & 0 & -\frac{(\rho-1)(\rho_F-\rho_H)\sigma_f}{1+2\rho\rho_F\rho_H} \\ 0 & \frac{\gamma_H-\gamma_F}{\sigma_\delta} & -\frac{(\rho-1)(\rho_H-\rho_F)\sigma_f}{1+2\rho\rho_F\rho_H} \\ \frac{2\widehat{g}_{A,t}(\gamma_F-\gamma_H)}{\sigma_\delta} & 0 & -\frac{2\widehat{g}_{A,t}(\rho-1)(\rho_H-\rho_F)}{1+2\rho\rho_F\rho_H}\sigma_f \\ 0 & \frac{2\widehat{g}_{B,t}(\gamma_H-\gamma_F)}{\sigma_\delta} & -\frac{2\widehat{g}_{B,t}(\rho-1)(\rho_H-\rho_F)}{1+2\rho\rho_F\rho_H}\sigma_f \\ \frac{\widehat{g}_{B,t}(\gamma_F-\gamma_H)}{\sigma_\delta} & \frac{\widehat{g}_{A,t}(\gamma_H-\gamma_F)}{\sigma_\delta} & -\frac{(\rho-1)(\rho_F-\rho_H)\sigma_f}{1+2\rho\rho_F\rho_H}\left(\widehat{g}_{B,t}-\widehat{g}_{A,t}(\rho_H-\rho_F)\right) \end{bmatrix} \tag{96}$$

6.6 Difference of Beliefs: Transform Analysis

We first observe that $\beta_1(t) = \varepsilon_A$, $\beta_2(t) = \varepsilon_B$, $\beta_5(t) = \chi$ such that the solution of the moment generating function is of the form

$$H(X, u-t|\varepsilon_A, \varepsilon_B, \chi) = \delta_A^{\varepsilon_A}\delta_B^{\varepsilon_B}\eta^\kappa H_{fg}\left(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_A, \widehat{g}_{B,t}\right) \tag{97}$$

where

$$\begin{aligned} H_{fg}\left(\widehat{f}^B_{A,t}, \widehat{f}^B_{B,t}, \widehat{g}_A, \widehat{g}_{B,t}\right) &= \exp\begin{pmatrix} \alpha(u-t) + \beta_3(u-t)\widehat{f}^B_{A,t} + \beta_4(u-t)\widehat{f}^B_{B,t} \\ +\beta_6(u-t)\widehat{g}_{A,t} + \beta_7(u-t)\widehat{g}_{B,t} \end{pmatrix} \\ &\quad\times \exp(\beta_8(u-t)\widehat{g}^2_{A,t} + \beta_9(u-t)\widehat{g}^2_{B,t} + \beta_{10}(u-t)\widehat{g}_{A,t}\widehat{g}_{B,t}). \end{aligned} \tag{98}$$

The functions of time β_3, β_4, β_6, β_7, β_8, β_9 and β_{10} are obtained as elements of the matrix

$$Z = \begin{pmatrix} 0 & \beta_3/2 & 0 & 0 \\ \beta_3/2 & \gamma & \beta_6/2 & \beta_7/2 \\ 0 & \beta_6/2 & \beta_8 & \beta_{10}/2 \\ 0 & \beta_7/2 & \beta_{10}/2 & \beta_9 \end{pmatrix}$$

where γ is an arbitrary function of time whose differential equation is mechanically provided by the system of matrix Riccati equations to be written and whose solution is completely irrelevant to the present model. We can write the following matrix Riccati equation

$$\dot{Z} = J + R^\top Z + ZR + ZQZ \tag{99}$$

where

$$
\begin{aligned}
J &= \begin{bmatrix} 0 & \frac{\varepsilon_A}{2} & 0 & 0 \\ \frac{\varepsilon_A}{2} & 0 & \frac{-\varepsilon_A \chi}{2} & \frac{-\varepsilon_B \chi}{2} \\ 0 & \frac{-\varepsilon_A \chi}{2} & \frac{\chi(\chi-1)}{2\sigma_\delta^2} & 0 \\ 0 & \frac{-\varepsilon_B \chi}{2} & 0 & \frac{\chi(\chi-1)}{2\sigma_\delta^2} \end{bmatrix}, \\
R &= \begin{bmatrix} -\zeta & 0 & -\frac{\chi(\varepsilon_A\gamma_F+\varepsilon_B\gamma_{HF})}{\varepsilon_A\sigma_\delta^2} & -\frac{\chi(\varepsilon_B\gamma_H+\varepsilon_A\gamma_{HF})}{\varepsilon_A\sigma_\delta^2} \\ 0 & 0 & 0 & 0 \\ 0 & \varepsilon_A(\gamma_F-\gamma_H) & -\zeta+\frac{-\chi\gamma_F+(\chi-1)\gamma_H}{\sigma_\delta^2} & -\frac{\gamma_{HF}}{\sigma_\delta^2} \\ 0 & \varepsilon_B(\gamma_H-\gamma_F) & -\frac{\gamma_{HF}}{\sigma_\delta^2} & -\zeta+\frac{-\chi\gamma_H+(\chi-1)\gamma_F}{\sigma_\delta^2} \end{bmatrix}, \\
Q &= \begin{bmatrix} 0 & 0 & a & b \\ 0 & 0 & 0 & 0 \\ a & 0 & \frac{2(\rho-1)^2(\rho_F-\rho_H)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2}+\frac{2(\gamma_F-\gamma_H)^2}{\sigma_\delta^2} & \frac{-2(\rho-1)^2(\rho_F-\rho_H)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2} \\ b & 0 & \frac{-2(\rho-1)^2(\rho_F-\rho_H)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2} & \frac{2(\rho-1)^2(\rho_F-\rho_H)^2\sigma_f^2}{(1+2\rho\rho_F\rho_H)^2}+\frac{2(\gamma_F-\gamma_H)^2}{\sigma_\delta^2} \end{bmatrix}.
\end{aligned}
$$

with

$$a = \frac{2\left(\frac{(\rho-1)(\rho_F-\rho_H)(\varepsilon_B(\rho\rho_F+\rho_H)+\varepsilon_A(\rho_F+\rho\rho_H))}{(1+2\rho\rho_F\rho_H)^2}\sigma_f^2+\frac{(\gamma_F-\gamma_H)(\varepsilon_A\gamma_F+\varepsilon_B\gamma_{HF})}{\sigma_\delta^2}\right)}{\varepsilon_A}$$

and

$$b = \frac{2\left(\frac{(\rho-1)(\rho_F-\rho_H)(\varepsilon_B(\rho\rho_F+\rho_H)+\varepsilon_A(\rho_F+\rho\rho_H))}{(1+2\rho\rho_F\rho_H)^2}\sigma_f^2-\frac{(\gamma_F-\gamma_H)(\varepsilon_B\gamma_H+\varepsilon_A\gamma_{HF})}{\sigma_\delta^2}\right)}{\varepsilon_A}.$$

Although β_3 and β_4 are trivially obtained (refer to the benchmark case), a closed-form solution to the latter matrix Riccati equation system requires them to be included. Indeed, if we had only considered the system of matrix Riccati equation associated with β_6, β_7, β_8, β_9 and β_{10}, a solution would have been obtained only up to a Magnus series approximation because the coefficient matrix would not have been constant. Instead, our solution is analytical and exact.

Reminiscent of Radon's lemma, we introduce the following change of variable $Z(u-t) = Y(u-t)X(u-t)^{-1}$. Then, we have that

$$\dot{Y} - Z\dot{X} = JX + R^\top Y + ZRX + ZQY.$$

Hence, by separation of variables, the matrices $X(u-t)$ and $Y(u-t)$ are the unique solutions of the linear Cauchy problem

$$
\begin{aligned}
\dot{X} &= -RX - QY,\ X(0) = I, \\
\dot{Y} &= JX + R^\top Y,\ Y(0) = 0,
\end{aligned}
\tag{100}
$$

where I is the 4×4 identity matrix. This system may be rewritten as

$$\frac{d}{dt}\begin{bmatrix} X(t) & Y(t) \end{bmatrix} = \begin{bmatrix} X(t) & Y(t) \end{bmatrix}\begin{bmatrix} -R & J \\ -Q & R^\top \end{bmatrix} \tag{101}$$

whose solution is of the form

$$\left[\begin{array}{cc} X(t) & Y(t) \end{array}\right] = \left[\begin{array}{cc} X(0) & Y(0) \end{array}\right] \exp\left(\left[\begin{array}{cc} -R & J \\ -Q & R^\top \end{array}\right] t\right). \tag{102}$$

(102) involves the matrix exponential function applied to a 8×8 matrix. Since this operation is expectated to be tedious, we prefer to consider the Jordan decomposition D of the matrix $M = \left[\begin{array}{cc} -R & J \\ -Q & R^\top \end{array}\right]$. This allows to rewrite (102) as

$$\left[\begin{array}{cc} X(t) & Y(t) \end{array}\right] = \left[\begin{array}{cc} X(0) & Y(0) \end{array}\right] S \exp\left(J_o t\right) S^{-1} \tag{103}$$

where J_0 is the Jordan matrix and S is a so-called similarity matrix pertaining to the Jordan decomposition of M. (103) is sufficient to pin down each and every β involved in H_{fg} except α.

6.7 Difference of Beliefs: Diffusion Matrix and Portfolio Holdings

6.7.1 Diffusion matrix

The diffusion matrix is obtained as in the benchmark case by premultiplying the diffusion of the state vector with the gradient of the stock prices:

$$\begin{aligned}
\Sigma_{1,t} &= \left[\begin{array}{ccccccc} \frac{\partial S_{A,t}}{\partial \delta_{A,t}} & \frac{\partial S_{A,t}}{\partial \delta_{B,t}} & \frac{\partial S_{A,t}}{\partial \widehat{f}_{A,t}} & \frac{\partial S_{A,t}}{\partial \widehat{f}_{B,t}} & \frac{\partial S_{A,t}}{\partial \eta_t} & \frac{\partial S_{A,t}}{\partial \hat{g}_{A,t}} & \frac{\partial S_{A,t}}{\partial \hat{g}_{B,t}} \\ \frac{\partial S_{B,t}}{\partial \delta_{A,t}} & \frac{\partial S_{B,t}}{\partial \delta_{B,t}} & \frac{\partial S_{B,t}}{\partial \widehat{f}_{A,t}} & \frac{\partial S_{B,t}}{\partial \widehat{f}_{B,t}} & \frac{\partial S_{B,t}}{\partial \eta_t} & \frac{\partial S_{B,t}}{\partial \hat{g}_{A,t}} & \frac{\partial S_{B,t}}{\partial \hat{g}_{B,t}} \end{array}\right] \times \\
&\times \left[\begin{array}{ccc} \sigma_\delta \delta_{A,t} & 0 & 0 \\ 0 & \sigma_\delta \delta_{B,t} & 0 \\ \frac{\gamma_F}{\sigma_\delta} & \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\rho_F+\rho\rho_H}{1+2\rho\rho_H\rho_F}\sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\gamma_H}{\sigma_\delta} & \frac{\rho_H+\rho\rho_F}{1+2\rho\rho_H\rho_F}\sigma_f \\ -\frac{\hat{g}_{A,t}}{\sigma_\delta}\eta_t & -\frac{\hat{g}_{B,t}}{\sigma_\delta}\eta_t & 0 \\ \frac{\gamma_F-\gamma_H}{\sigma_\delta} & 0 & -\frac{(\rho-1)(\rho_F-\rho_H)}{1+2\rho\rho_H\rho_F}\sigma_f \\ 0 & \frac{\gamma_H-\gamma_F}{\sigma_\delta} & -\frac{(\rho-1)(\rho_H-\rho_F)}{1+2\rho\rho_H\rho_F}\sigma_f \end{array}\right] \\
&= \left[\begin{array}{ccc} \sigma_{S_A}^{\delta_A} & \sigma_{S_A}^{\delta_B} & \sigma_{S_A}^{s} \\ \sigma_{S_B}^{\delta_A} & \sigma_{S_B}^{\delta_B} & \sigma_{S_B}^{s} \end{array}\right].
\end{aligned}$$

6.7.2 Exposures

$$
\begin{aligned}
E_{B,t} &= \left[\begin{array}{ccccccc} \frac{\partial W_t^B}{\partial \delta_{A,t}} & \frac{\partial W_t^B}{\partial \delta_{B,t}} & \frac{\partial W_t^B}{\partial \widehat{f}_{A,t}} & \frac{\partial W_t^B}{\partial \widehat{f}_{B,t}} & \frac{\partial W_t^B}{\partial \eta_t} & \frac{\partial W_t^B}{\partial \widehat{g}_{A,t}} & \frac{\partial W_t^B}{\partial \widehat{g}_{B,t}} \end{array} \right] \times \\
& \times \left[\begin{array}{ccc} \sigma_\delta \delta_{A,t} & 0 & 0 \\ 0 & \sigma_\delta \delta_{B,t} & 0 \\ \frac{\gamma_F}{\sigma_\delta} & \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\rho_F+\rho\rho_H}{1+2\rho\rho_H\rho_F}\sigma_f \\ \frac{\gamma_{HF}}{\sigma_\delta} & \frac{\gamma_H}{\sigma_\delta} & \frac{\rho_H+\rho\rho_F}{1+2\rho\rho_H\rho_F}\sigma_f \\ -\frac{\hat{g}_{A,t}}{\sigma_\delta}\eta_t & -\frac{\hat{g}_{B,t}}{\sigma_\delta}\eta_t & 0 \\ \frac{\gamma_F-\gamma_H}{\sigma_\delta} & 0 & -\frac{(\rho-1)(\rho_F-\rho_H)}{1+2\rho\rho_H\rho_F}\sigma_f \\ 0 & \frac{\gamma_H-\gamma_F}{\sigma_\delta} & -\frac{(\rho-1)(\rho_H-\rho_F)}{1+2\rho\rho_H\rho_F}\sigma_f \end{array} \right] \\
&= \left[\begin{array}{ccc} E_{B,t}^{\delta_A} & E_{B,t}^{\delta_B} & E_{B,t}^{s} \end{array} \right].
\end{aligned}
$$

The optimal 1x3 portfolio vector held by investor located in country B solves

$$
\left[\begin{array}{ccc} E_{B,t}^{\delta_A} & E_{B,t}^{\delta_B} & E_{B,t}^{s} \end{array} \right] = \left[\begin{array}{ccc} \theta_{S_A,t}^B & \theta_{S_B,t}^B & \theta_{F_s,t}^B \end{array} \right] \times \left[\begin{array}{ccc} \sigma_{S_A}^{\delta_A} & \sigma_{S_A}^{\delta_B} & \sigma_{S_A}^{s} \\ \sigma_{S_B}^{\delta_A} & \sigma_{S_B}^{\delta_B} & \sigma_{S_B}^{s} \\ 0 & 0 & 1 \end{array} \right].
$$

Here, we used the same technique as in Dumas, Lewis and Osambela (2008) to construct the diffusion matrix by parts. We added a line to the diffusion matrix previously derived in order to account for the presence of the futures contract contained in the menu of assets. Since the latter asset is only marked to the signal and that its variance is normalized to 1, the third line follows. Also, the exposures are obtained by premultiplying the diffusion matrix by the gradient of Agent B's wealth.

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