



FINANCIAL DERIVATIVES

FINE 448, SECTION 002 (Instructor: Daniel Andrei)

WEDNESDAY, DECEMBER 17, 2025, 9:00AM (DURATION: 3 HOURS)

EXAMINER: Daniel Andrei

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STUDENT NAME:		McGILL ID:												
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EXAM:	CLOSED BOOK <input type="checkbox"/> OPEN BOOK <input checked="" type="checkbox"/>
	SINGLE-SIDED <input type="checkbox"/> PRINTED ON BOTH SIDES OF THE PAGE <input checked="" type="checkbox"/>
	MULTIPLE CHOICE ANSWER SHEETS: YES <input type="checkbox"/> NO <input checked="" type="checkbox"/> <small>NOTE: The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 17 of the Code of Student Conduct and Disciplinary Procedures.</small>
	ANSWER BOOKLET REQUIRED: YES <input type="checkbox"/> NO <input checked="" type="checkbox"/>
	EXTRA BOOKLETS PERMITTED: YES <input checked="" type="checkbox"/> NO <input type="checkbox"/>
	ANSWER ON EXAM: YES <input checked="" type="checkbox"/> NO <input type="checkbox"/>
	SHOULD THE EXAM BE: RETURNED <input checked="" type="checkbox"/> KEPT BY STUDENT <input type="checkbox"/>
CRIB SHEETS:	PERMITTED <input checked="" type="checkbox"/> <u>Specifications:</u> (ex: one 8 1/2X11 handwritten double-sided sheet) NOT PERMITTED <input type="checkbox"/>
DICTIONARIES:	TRANSLATION ONLY <input type="checkbox"/> REGULAR <input checked="" type="checkbox"/> NOT PERMITTED <input type="checkbox"/>
CALCULATORS:	NOT PERMITTED <input type="checkbox"/> PERMITTED (Non-Programmable) <input type="checkbox"/> PERMITTED (Programmable) <input checked="" type="checkbox"/>
ANY SPECIAL INSTRUCTIONS: e.g. molecular models	FULL DOCUMENTATION ALLOWED. Students CANNOT use a laptop, tablet, phone, or any other communication device. Students are NOT allowed to communicate with anyone during the exam. Questions are NOT allowed. Students must answer on the exam (NO multiple choice answer sheet needed). Only one answer is valid per question. Due to rounding, your answers may be slightly different from the choices you are given. Generally, always pick the choice that is closest to your calculated answer.

1 (0.5 points) A European call and a European put are written on the same non-dividend-paying asset (S_0). Both options have the same maturity and are struck at-the-money ($K = S_0$). If the risk-free rate is strictly positive ($r > 0$), the call's value is greater than the put's value.

- a. True
- b. False

2 (0.5 points) Let Ψ_P be the Psi per percentage point of a European put written on the underlying S_0 , with strike K and maturity T . Consider a European call written on the same underlying, with the same strike and the same maturity. The Psi of the call option (per percentage point), Ψ_C , equals

$$\Psi_C = \Psi_P - S_0 T e^{-\delta T}$$

- a. True
- b. False

3 (0.5 points) Let ρ_P be the Rho per percentage point of a European put written on the underlying S_0 , with strike K and maturity T . Consider a European call written on the same underlying, with the same strike and the same maturity. The Rho of the call option (per percentage point), ρ_C , equals

$$\rho_C = \rho_P + K T e^{-r T}$$

- a. True
- b. False

4 (0.5 points) An asset trades at $S_0 = \$45$. Consider two portfolios of European options with the same maturity:

Portfolio 1: A long call with strike $K_1 = \$40$, short two calls with strike $K_2 = \$45$, and long a call with strike $K_3 = \$50$.

Portfolio 2: A long call with strike $K_1 = \$40$ and a short call with strike $K_3 = \$50$.

Portfolio 1 has a higher value than Portfolio 2.

- a. True
- b. False

5 (0.5 points) A non-dividend-paying stock has an expected return of $\mu = 12\%$. The risk-free rate is $r = 5\%$. An investor writes a European put option and creates a delta-neutral portfolio by trading the underlying stock. The expected return on this delta-hedged portfolio is 12%.

- a. True

b. False

6 (0.5 points) Consider an asset that pays no dividends ($\delta = 0$), and let the risk-free interest rate be zero ($r = 0$). A call calendar spread is built by selling a call with maturity T_1 and buying a call with maturity T_2 ($T_2 > T_1$), both with strike K .

This portfolio's value is exactly the same as a put calendar spread (selling $P(T_1, K)$ and buying $P(T_2, K)$) with the same strikes and maturities.

(Hint: Recall the Put-Call Parity relationship $C - P = S_0 e^{-\delta T} - K e^{-rT}$ and apply the conditions.)

a. True

b. False

7 (0.5 points) The Theta of a plain vanilla European call option on a non-dividend paying asset is always negative.

a. True

b. False

8 (0.5 points) The magnitude of time decay (Theta) for a plain vanilla European option is greatest when the option is *at-the-money* (ATM) and close to expiration.

a. True

b. False

9 (0.5 points) The Elasticity of an out-of-the-money call is greater than the elasticity of an in-the-money call.

a. True

b. False

10 (0.5 points) The mathematical model for a delta-gamma hedge approximates the change in an option's price using a Taylor series expansion with respect to the stock price, truncated after the second-order term (Gamma).

a. True

b. False

11 (0.5 points) Consider a non-dividend paying stock. The delta of an at-the-money European call in the Black-Scholes model is at most $1/2$.

a. True

b. False

12 (0.5 points) The delta of a straddle is a positive number that takes values between 0 and 1.

a. True

b. False

13 (0.5 points) Let θ_P be the per-day Theta of a European put written on the underlying S_0 , with strike K and maturity T . Consider a European call written on the same underlying, with the same strike and the same maturity. The per-day Theta of the call option, θ_C , equals

$$\theta_C = \theta_P + \frac{\delta S_0 e^{-\delta T} - r K e^{-rT}}{365}$$

a. True

b. False

14 (0.5 points) Let $S = 100$, $K = 100$, $r = 0.05$, and $\delta = 0.05$. Then European calls and puts with the same maturity T will always have the same price, irrespective of the maturity T or of the volatility of the underlying σ .

a. True

b. False

15 (0.5 points) Consider a non-dividend paying asset. The risk-free rate is 1%. The value of a European call option written on this asset is always higher than the call's intrinsic value.

a. True

b. False

16 (0.5 points) Consider an asset with strictly positive beta. The beta of a deep out-of-the-money European call option written on this asset can be lower than the beta of the asset.

a. True

b. False

17 (0.5 points) The delta of a straddle composed of two options with delta-symmetric strike is zero.

a. True

b. False

18 (0.5 points) Consider European options on a non-dividend paying asset with maturity T . The put-call parity relationship, expressed in terms of the forward price $F_{0,T}$, can be written as:

$$e^{rT}(C_0 - P_0) = F_{0,T} - K$$

a. True

b. False

19 (0.5 points) The risk-neutral probability that the stock price ends up above its mean at maturity is always lower than 50%.

a. True

b. False

20 (0.5 points) The delta of a standard long butterfly spread, constructed with calls, must always be positive or zero.

a. True

b. False

21 (1 point) What is the formula for calculating the profit of an investor who writes a *covered call* (buying a non-dividend paying stock and selling a European call option with maturity T on the same stock)? *At the purchase (today), the stock price, S_0 , equals the strike price of the option, K . Assume that $S_T > K$ at maturity.* The interest rate is r , and the call premium today is C_0 .

a. The profit at maturity is $K(1 - e^{rT}) - C_0e^{rT}$

b. The profit at maturity is $K(1 - e^{rT}) + C_0e^{rT}$

c. The profit at maturity is $K - S_0 + C_0e^{rT}$

d. The profit at maturity is $K(1 - e^{-rT}) + C_0e^{-rT}$

22 (1 point) If the value of the assets of a firm is \$1 billion, its volatility is 20%/year, the face value of debt is \$700 million due in 1 year, the risk free rate is 5%, and the stock as an option on the firm has $N(d_1) = 0.98$ and $N(d_2) = 0.97$, what is the risk neutral probability that the firm defaults on its debt obligation in 1 year?

a. 97%

b. 4%

- c. 3%
- d. 2%

23 (1 point) Assume the following Black-Scholes parameters for a stock and an European call option on it:

Price of the underlying	S_0
Strike price	K
Volatility of the underlying	σ
Interest rate (cont. compounded)	r
Maturity of the option	T_1
Dividend yield of the underlying	δ

Consider a futures contract on the stock with maturity $T_2 > T_1$ and an European call option **on the futures**. The option has maturity T_1 and strike price K . What are the Black-Scholes parameters used to compute this option's price?

(Hint: A futures can be viewed as an asset whose dividend yield is such that its expected growth rate is zero.)

- a. Price of the underlying: $S_0e^{(r-\delta)T_2}$; Strike price: K ; Volatility of the underlying: σ ; Interest rate (cont. compounded): r ; Maturity of the option: T_1 ; Dividend yield of the underlying: δ .
- b. Price of the underlying: $S_0e^{(r-\delta)T_2}$; Strike price: K ; Volatility of the underlying: σ ; Interest rate (cont. compounded): r ; Maturity of the option: T_1 ; Dividend yield of the underlying: r .
- c. Price of the underlying: $S_0e^{(r-\delta)T_2}$; Strike price: K ; Volatility of the underlying: σ ; Interest rate (cont. compounded): r ; Maturity of the option: T_1 ; Dividend yield of the underlying: 0.
- d. Price of the underlying: $S_0e^{(r-\delta)T_2}$; Strike price: K ; Volatility of the underlying: σ ; Interest rate (cont. compounded): r ; Maturity of the option: T_1 ; Dividend yield of the underlying: $\delta - r$.

24 (1 point) Company ZED pays no dividends. Its stock price is \$30. The three-month European call at strike 29 is trading at \$3. What is the price of the European put with the same strike and the same maturity?

- a. \$1.93
- b. \$1.30
- c. I don't have enough information to answer this question
- d. \$3

25 (1 point) Below is some information about two options written on the same underlying. Suppose you buy five 45-strike calls, one 45-strike put, and 0.121 shares of the underlying, whose price is 40.

Strike=45	Call	Put
Price	3.4042	7.4679
Delta	0.433	-0.5539
Gamma	0.0309	0.0309
Vega	0.1632	0.1632
Theta	-0.0067	-0.0044
Rho	0.1531	-0.3259
Elasticity	5.0881	-2.9667

What is the elasticity of your portfolio?

- The elasticity of the portfolio is 2.3625.
- The elasticity of the portfolio is 0.0004.
- The elasticity of the portfolio is 1.1234.
- The elasticity of the portfolio is 4.5678.

26 (1 point) Assume you own a portfolio consisting of Call 1, Call 2, and Call 3 from the table below. You want to make this portfolio *delta-gamma neutral* by adding a position in Call 4 and in the underlying stock.

	Call 1	Call 2	Call 3	Call 4
Number of contracts	450	100	-400	-300
Days to maturity	120	60	60	30
Strike	105	115	100	100
Call Price	4.99	0.58	4.88	3.27
Delta	0.4843	0.1218	0.5843	0.5598
Gamma	0.0278	0.0199	0.0385	0.055

How many units of Call 4 and how many units of the stock must be traded?

- Buy 16.03 units of Call 4. Sell 5.38 units of the underlying.
- Sell 38.53 units of Call 4. Buy 102.33 units of the underlying.
- Buy 38.53 units of Call 4. Sell 102.33 units of the underlying.
- Sell 16.03 units of Call 4. Buy 5.38 units of the underlying.

27 (1 point) Assume you own the portfolio of European options shown in the table below

	Put 1	Put 2	Put 3
Number of contracts	100	-200	100
Years to maturity	1	1	1
Strike	20	30	40
Put Price	0.1587	2.4571	8.5604
Delta	-0.038	-0.3351	-0.7025
Gamma	0.0089	0.0391	0.0372
Vega	0.0256	0.113	0.1074
Theta	-0.0009	-0.0029	-0.0003
Rho	-0.0134	-0.1285	-0.3034

Suppose that the position is held over the next 10 days. If during this time period the underlying asset price decreases by \$1, what is the approximate gain or loss of the portfolio over the next 10 days?

- The portfolio makes a gain of \$11.66.
- The portfolio makes a loss of \$11.66.
- The portfolio makes a gain of \$7.01.
- The portfolio makes a gain of \$26.93.

28 (1 point) Assume you own the portfolio of European options shown in the table below

	Put 1	Put 2	Put 3
Number of contracts	100	-200	100
Years to maturity	1	1	1
Strike	20	30	40
Put Price	0.1587	2.4571	8.5604
Delta	-0.038	-0.3351	-0.7025
Gamma	0.0089	0.0391	0.0372
Vega	0.0256	0.113	0.1074
Theta	-0.0009	-0.0029	-0.0003
Rho	-0.0134	-0.1285	-0.3034

Suppose you want to make this portfolio **vega-neutral** by **modifying only your position in Put 1** (the one with strike 20). What should your new, final position in Put 1 be?

- The new position should be 287.45 units.
- The new position should be -363.28 units.
- The new position should be 463.28 units.
- The new position should be -9.30 units.

29 (1 point) You want to use European options written on a stock to capture the dividends that the stock pays until the maturity T of the options. What is the approach of implementing this strategy?

(Hint: Recall the Put-Call Parity relationship $C_0 - P_0 = S_0 - PV(Div) - PV(K)$.)

- The only way to capture a stock's dividend is to buy the stock itself, and not by holding any positions in options.
- Sell a put option with maturity T and strike K , buy a call option with the same strike and the same maturity, sell the stock, and lend the present value of K .
- Buy a put option with maturity T and strike K , sell a call option with the same strike and the same maturity, buy the stock, and borrow the present value of K .
- Buy a put option with maturity T and strike K , sell a call option with the same strike and the same maturity, sell the stock, and borrow the present value of K .

30 (1 point) Consider two European options, one put and one call, written on the same underlying S_0 . Both options have the same maturity T and the same strike K . The risk-free interest rate (continuously compounded) is r , and the dividend yield of the underlying is δ . *The options have the same price today.* Write the strike of these options as a function of all the **other** parameters provided above.

- I do not have enough information to answer this question.
- $K = S_0 e^{(r-\delta)T}$
- $K = S_0 e^{-\delta T}$
- $K = S_0 e^{-(r-\delta)T}$

31 (1 point) In a calendar spread, the Delta of the long-maturity option is 0.3976, and the Delta of the short-maturity option is 0.2869. To delta-hedge this calendar spread, you will have to

- Short 0.1107 units of the underlying
- Sell the short-maturity option
- Buy 0.1107 units of the underlying
- The calendar spread is already Delta-hedged

32 (1 point) Consider a non-dividend paying stock. What happens to the Delta of an at-the-money European call as time to maturity declines?

(Hint: Think about what d_1 simplifies to when $S_0 = K$, and then ask yourself what value d_1 must approach as T gets closer and closer to 0.)

- a. I don't have enough information to answer this question.
- b. The Delta of the at-the-money call decreases as time to maturity declines.
- c. Since this is an at-the-money option, its Delta remains unchanged as time to maturity declines.
- d. The Delta of the at-the-money call increases as time to maturity declines.

33 (1 point) What is the two-year volatility of an asset with 25% three-month volatility?

- a. The two-year volatility is 84.85%
- b. The two-year volatility is 100%
- c. The two-year volatility is 70.71%
- d. The two-year volatility is 200%

34 (1 point) What does the Black-Scholes option pricing model assume about the probability distribution of the stock price at the maturity of the option?

- a. The stock price follows a Poisson distribution
- b. The logarithm of the stock price follows a lognormal distribution
- c. The stock price follows a normal distribution
- d. The stock price follows a lognormal distribution

35 (1 point) In a calendar spread, the Vega of the long-maturity option is 0.1905, and the Vega of the short-maturity option is 0.0975. If the volatility of the returns of the underlying goes up by 1 percentage point (e.g., from 30% to 31%), the calendar spread:

- a. Increases by 9.3%
- b. Increases by 9.3 cents
- c. The calendar spread is Vega-neutral
- d. Decreases by 9.3 cents

36 (1.5 points) An at-the-money European call on an underlying asset paying dividends continuously at rate δ is worth more than an at-the-money European put with the same maturity if and only if

- a. $\delta > r$
- b. $r = 0$
- c. $T = 1$

d. $\delta < r$

37 (1.5 points) An **asset-or-nothing put option** gives you the right to own one share of the underlying if at maturity the share price drops below \$100. *The price of this put option today is \$30.* Consider an **asset-or-nothing call option** with the same maturity and the same strike price as the above put option. It gives you the right to own one share of the underlying if at maturity the share price exceeds \$100.

The share price today is \$95, and the stock pays no dividends. Write down the put-call parity relation at time 0 (today) between these two asset-or-nothing options. What is today's price of the asset-or-nothing call option?

- a. The put-call parity is $C_0 - P_0 = S_0 - Ke^{-rT}$. To compute the price of the call, I need the maturity and the interest rate.
- b. The put-call parity is $C_0 + P_0 = S_0$. The price of the call is \$65.
- c. The put-call parity is $C_0 + P_0 = S_0 - Ke^{-rT}$. To compute the price of the call, I need the maturity and the interest rate.
- d. The put-call parity is $C_0 - P_0 = S_0 - 100$. The price of the call is \$35.

38 (1.5 points) Consider a firm whose stock price today of $S_0 = \$100$. This firm does not pay dividends. When trying to compute the price of a European put option, you found $d_1 = 0$ and $P_0 = \$11.7711$. If the stock price decreases by 1% today, how does the put price change?

- a. The put price increases by 4.25%
- b. I do not have enough information to answer this question.
- c. The put price increases by 42.5 cents
- d. The put price increases by 5.82%

39 (1.5 points) A European put option on a non-dividend paying stock with a strike price of 1,000 and a 6-month expiration costs 74.20 today. The effective interest rate over the 6-month period is 8%. What is the stock price after 6 months that would make the profit from holding a long position in this put option equal to the profit from holding a short position in the same put option?

- a. $S_T = 922.77$.
- b. $S_T = 919.86$.
- c. There is no possible stock price after 6 months that would make the profit from holding a long position in this put option equal to the profit from holding a short position in the same put option.
- d. $S_T = 1000$.

40 (1.5 points) Compute the elasticity of a long straddle position that consists of the following options:

Option	Price	Elasticity
Call	7.9234	7.1076
Put	6.6164	-6.5525

- The elasticity of the long straddle position is 0.3451.
- The elasticity of the long straddle position is 89.15.
- The elasticity of the long straddle position is 0.8915.
- The elasticity of the long straddle position is 34.51.

41 (1.5 points) You have bought a calendar spread using the two call options below. You calculate the spread's greeks and find it has a **negative gamma**. You are concerned that the negative gamma will make your position's Delta unstable and lead to accelerating losses if the stock price moves significantly. To solve this, you decide to make the portfolio **gamma-neutral** by trading the available put option.

Asset	Delta	Gamma
Call option, long maturity	0.5548	0.0281
Call option, short maturity	0.5351	0.0448
Put option (hedging)	-0.3848	0.0136

What position in the put option is required to make your portfolio gamma-neutral?

- Sell 1.2279 put options
- Sell 0.4541 put options
- Buy 1.2279 put options
- Buy 0.4541 put options

42 (1.5 points) We know from the put-call parity that

$$C_0 - P_0 = S_0 e^{-\delta T} - K e^{-rT}$$

Consider a European call option with strike \$100 and maturity 1 year. The Vega of this call option is 0.30. What is the Vega of the European put with the same strike and the same maturity?

(Hint: Differentiate the put-call parity equation with respect to σ .)

- 0.4
- 0.3

- c. 0.35
- d. 0.25

43 (1.5 points) Consider a firm whose stock price today of $S_0 = \$100$. This firm does not pay dividends. When trying to compute the price of a European put option, you found $d_1 = 0$ and $P_0 = \$11.7711$. If the stock price increases by 1% today, how does the put price change?

- a. The put price decreases by 4.25%
- b. The put price decreases by 5.82%
- c. I do not have enough information to answer this question.
- d. The put price decreases by 42.5 cents

44 (1.5 points) The price of an asset today is $S_0 = \$100$. A European put option written on this asset has a Delta of -0.4864 and an Elasticity of -6.4874. What is the price of the European put option?

- a. I do not have enough information to answer this question.
- b. The price of the European put option is 7.4976.
- c. The price of the European put option is 6.3560.
- d. The price of the European put option is 8.2023.

45 (1.5 points) You have bought a straddle written on an asset with dividend yield $\delta = 0.05$. The maturity of the straddle is $T = 1$. The Delta of the call option from this straddle is 0.5. What is the Delta of the straddle?

- a. The Delta of the straddle is 0.
- b. The Delta of the straddle is -0.0488.
- c. I do not have enough information to answer this question.
- d. The Delta of the straddle is 0.0488.

46 (2 points) What specific relationship between the risk-free rate (r), the dividend yield (δ), and the volatility (σ) is required for an at-the-money straddle (where $S_0 = K$) to have a net delta of exactly zero?

- a. An at-the-money straddle cannot possibly have a delta of zero.
- b. When $\delta = r + \sigma^2/2$
- c. When $\delta = r$

d. When $\sigma = 0$

47 (2 points) Let $S = 40$, $\sigma = 0.3$, $r = 0.05$, and $\delta = 0.02$. What is the strike for which a straddle with maturity $T = 1.5$ written on this asset has a maximum exposure to volatility, i.e., a maximum vega? Given that strike, what is the delta of the straddle?

(Hint: In Black–Scholes, vega is proportional to $N'(d_1)$.)

- a. The strike is 40.60. The delta of the straddle is 0.08.
- b. The strike is 41.22. The delta of the straddle is 0.12.
- c. The strike is 44.76. The delta of the straddle is 0.
- d. The strike is 41.53. The delta of the straddle is 0.

48 (2 points) *The prices of European options (calls or puts) are strictly convex functions of the strike.* Consider two European put options with strikes 40 and 50 on the same underlying asset and with the same maturity, trading for \$8 and \$10, respectively. What is the value of a put option with strike 0 on the same underlying asset and with the same maturity? Is there an arbitrage? If yes, which of the above two put options is relatively overpriced?

(Hint: Plot (K, P) at $K = 0, 40, 50$.)

- a. The value of the zero-strike put option is infinity. Yes, these prices create an arbitrage opportunity. The put with strike 40 is relatively overpriced.
- b. The value of the zero-strike put option is zero. It is not possible to know whether these prices create an arbitrage opportunity.
- c. The value of the zero-strike put option is zero. No, these prices do not create an arbitrage opportunity. No option is relatively overpriced.
- d. The value of the zero-strike put option is zero. Yes, these prices create an arbitrage opportunity. The put with strike 40 is relatively overpriced.

49 (2 points) Consider the following call options, used for a butterfly spread position:

Option	Price	Delta
Call (strike=98)	9.9385	0.9192
Call (strike=100)	8.2235	0.8674
Call (strike=102)	6.6359	0.7979

The maturity of the options is 0.5 years, and the interest rate is 5.5%, continuously compounded. What is the risk-neutral probability of the underlying reaching values at maturity between 98 and 102? What is the delta of the butterfly spread?

- a. The risk-neutral probability of the underlying reaching values at maturity between 98 and 102 is 13.10%. The delta of the butterfly spread is 0.0042.

- b. The risk-neutral probability of the underlying reaching values at maturity between 98 and 102 is 5.24%. The delta of the butterfly spread is -0.0177.
- c. The risk-neutral probability of the underlying reaching values at maturity between 98 and 102 is 5.24%. The delta of the butterfly spread is 0.0110.
- d. The risk-neutral probability of the underlying reaching values at maturity between 98 and 102 is 13.10%. The delta of the butterfly spread is -0.0177.

50 (2 points) Consider a structured investment product that is available from Credit Suisse for an initial cost of \$1 and whose payoff in 3 years is given by

$$1 + \max \left[\alpha, c \left(\frac{S_3}{S_0} - 1 \right) \right]$$

where α , c , are positive constants and S_t denotes the level of the S&P500 at time t . The risk-free interest rate is 8%, the volatility of the S&P500 is equal to 15% per year and the dividend yield on the index is $\delta = 3\%$ per year.

Assume that Credit Suisse offers you to invest in a contract with $c = 0.75$ and $\alpha = 15\%$. What is the price of this contract? Would you pay \$1 for this structured product? What is the price of the contract if the volatility was equal to 35%?

- a. I don't have enough information to answer this question
- b. The price of the contract is 0.9659. I wouldn't invest in this product. If the volatility equals 35%, the price of the contract becomes 1.0435.
- c. The price of the contract is 0.9659. I wouldn't invest in this product. If the volatility equals 35%, the price of the contract becomes 1.0595.
- d. The price of the contract is 1.0595. Yes, I would invest in this product. If the volatility equals 35%, the price of the contract becomes 0.9659.

51 (2 points) Let $S = 100$, $\sigma = 0.50$, $r = 0.08$, $T = 1$, and $\delta = 0$. In the Black-Scholes framework, compute the risk-neutral probability that the stock's continuously compounded return over the year is greater than 10%.

- a. 0.5433
- b. 0.4566
- c. 0.3860
- d. 0.2845

52 (2 points) Firm A has a stock price of \$38 and has made an offer for firm B where A promises to pay \$60/share for B, as long as A's stock price remains between \$35 and \$45. If the price of A is below \$35, A will pay 1.714 shares, and if the price of A is above \$45, A will

pay 1.333 shares. The deal is expected to close in 9 months. Assume (for A's stock): $\sigma = 30\%$, $r = 1\%$, and $\delta = 0$.

Hint: The payoff of this offer can be replicated. You may use the following 9-month option data: A 35-strike call option has a price of \$5.61 and a delta of 0.683; a 45-strike call option has a price of \$1.74 and a delta of 0.310.

What is the value of the offer? How does the value of the offer change if the price of A's stock goes up by 1%?

- The value of the offer is \$57.83. If the price of A's stock goes up by 1%, then the value of the offer goes up by 0.63%
- The value of the offer is \$57.83. If the price of A's stock goes up by 1%, then the value of the offer does not change.
- The value of the offer is \$56.81. If the price of A's stock goes up by 1%, then the value of the offer goes up by 0.63%.
- The value of the offer is \$56.81. If the price of A's stock goes up by 1%, then the value of the offer goes up by 1%.

53 (2 points) Consider European call options that are exactly at-the-money ($S_0 = K$), with $\delta = 0$. You have a long-maturity option (T_L) and a short-maturity option (T_S), where $T_S < T_L$.

You want the risk-neutral probability of the long option finishing in-the-money to be *equal* to the probability of the short option finishing in-the-money. That is: $\text{Prob}(S_{T_L} > K) = \text{Prob}(S_{T_S} > K)$.

Under what condition is this possible?

- This is only possible if $r = \sigma^2/2$.
- This is always satisfied.
- This is only possible if $T_S = T_L/2$.
- This is impossible to satisfy.

54 (2 points) Let $V^+(K)$ denote the value of a bet with strike K that pays \$1 at time T if $S_T > K$:

$$V^+(K) = e^{-rT} N(d_2(K))$$

where $d_2(K)$ is the standard Black-Scholes d_2 parameter calculated with strike K .

What is the value V of a bet that pays \$1 at time T if $K_1 \leq S_T \leq K_2$? (Assume $K_1 < K_2$).

- $V = e^{-rT} [N(d_2(K_1)) - N(d_2(K_2))]$
- $V = e^{-rT} [N(d_2(K_2)) - N(d_2(K_1))]$

- c. $V = e^{-rT}[N(d_2(K_1)) + N(d_2(K_2))]$
 d. $V = e^{-rT}[N(d_1(K_1)) - N(d_1(K_2))]$

55 (2 points) If the delta of a European call on a non-dividend paying asset is 0.6, what is the delta of the European put for the same strike and maturity? What if the dividend yield on the asset is $\delta = 0.05$ and the maturity of the options is 1 year?

- a. In the first case, the delta of the put is -0.4. In the second case, the delta of the put is -0.3512.
 b. In the first case, the delta of the put is -0.4. In the second case, the delta of the put is -0.4512.
 c. In the first case, the delta of the put is 0.4. In the second case, the delta of the put is -0.4512.
 d. In the first case, the delta of the put is -0.4. In the second case, the delta of the put is 0.3512.

56 (2 points) Consider the following two portfolios of European options:

Portfolio 1: Long one call option with strike 95 and long one call option with strike 105. The value of this portfolio is \$10.

Portfolio 2: Long two call options with strike 100. The value of this portfolio is \$11.

(All options are on the same asset and have the same maturity.)

Is there an arbitrage opportunity? If yes, which one of the two portfolios is overvalued, and by how much?

- a. Yes, there is an arbitrage opportunity. The portfolio 1 is overvalued, by at least 1\$.
 b. It's impossible to tell whether there is an arbitrage opportunity.
 c. No, there is no arbitrage opportunity. No portfolio is overvalued.
 d. Yes, there is an arbitrage opportunity. The portfolio 2 is overvalued, by at least 1\$.

57 (2 points) You buy 1000 six months at-the-money call options on a non-dividend paying asset with spot price \$100, following a lognormal process with volatility 25%. Assume the interest rates are constant at 5%. How much you expect your portfolio to change if the stock price suddenly decreases to \$98? To avoid this risk, what delta-hedging position do you need to take?

- a. If the stock price decreases to \$98, I expect to lose \$1182. The position is delta-hedged by buying 590.88 units of the underlying asset.
 b. If the stock price decreases to \$98, I expect to lose \$1182. The position is delta-hedged by shorting 590.88 units of the underlying asset.

- c. If the stock price decreases to \$98, I expect to lose \$1523. The position is delta-hedged by shorting 590.88 units of the underlying asset.
- d. If the stock price decreases to \$98, I expect to lose \$987. The position is delta-hedged by buying 590.88 units of the underlying asset.

58 (2 points) You are considering the purchase of a 3-month 41.5-strike American call option on a non-dividend-paying stock. The Black-Scholes framework holds. The stock is currently selling for 40, and the stock's return volatility is 30%. The current call option delta is 0.5. What is the risk-free rate?

- a. The risk-free rate is 8.2256%.
- b. The risk-free rate is 10.226%.
- c. The risk-free rate is 4.5000%.
- d. Since this is an American option, we cannot apply the Black-Scholes formula to find r .

59 (2 points) Consider a straddle with strike $K = 100$, maturity $T = 1$, and let $\sigma = 0.3, r = 0.05$, and $\delta = 0$. What is the Delta of the straddle? For what price of the underlying you would have a straddle with Delta=0?

- a. The Delta of the straddle is $2N(d_1) - 1$. The Delta equals zero when $S = 91.9373$.
- b. The Delta of the straddle is $1 - 2N(d_1)$. The Delta equals zero when $S = 89.9373$.
- c. The Delta of the straddle is $2N(d_1) - 1$. The Delta equals zero when $S = 90.9373$.
- d. The Delta of the straddle is $1 - 2N(d_1)$. The Delta equals zero when $S = 91.9373$.

60 (2 points) The time until expiration for all options is 0.6 years and the underlying stock pays no dividends. Suppose you observe the following option prices:

Strike price	50	60
Call price	7.0140	2.8195
Put price	2.8139	8.1794

What transaction using all four of these options will enable you to buy the stock forward at \$50 and sell the stock forward at \$60? What is the continuously compounded interest rate?

- a. I don't have enough information to answer this question.
- b. Buy the 50 call, sell the 50 put, sell the 60 call, and buy the 60 put. The per annum continuously compounded interest rate is 4.5%.
- c. Sell the 50 call, buy the 50 put, buy the 60 call, and sell the 60 put. The per annum continuously compounded interest rate is 7.5%.

- d. Buy the 50 call, sell the 50 put, sell the 60 call, and buy the 60 put. The per annum continuously compounded interest rate is 7.5%.

61 (2 points) You are long 1000 call options with strike 90 and three months to maturity. The underlying asset price is \$92, volatility is $\sigma = 0.2$, the risk-free rate is $r = 0.1$, and $\delta = 0$.

The option's Delta is 0.6984 and its Vega is 0.1603.

What delta-hedging position do you need to take? What is the vega of your final (delta-hedged) portfolio?

- The position is delta-hedged by shorting 698.4 units of the underlying asset. The vega of the resulting portfolio is 160.3.
- The position is delta-hedged by shorting 698.4 units of the underlying asset. The vega of the resulting portfolio is -160.3 .
- The position is delta-hedged by shorting 698.4 units of the underlying asset. The vega of the resulting portfolio is 200.3.
- The position is delta-hedged by shorting 698.4 units of the underlying asset. The vega of the resulting portfolio is 0.

62 (2 points) Consider a binomial tree with $n = 100$ periods. The risk-neutral probability of an "up" movement is $p = 0.4875$. Using the convergence of the binomial distribution to the normal distribution, what is the risk-neutral probability of obtaining at most $k = 45$ "up" movements?

- 50.00%
- I don't have enough information to answer this question
- 48.55%
- 22.66%

(Hint: Standardize with $Z = \frac{k - np}{\sqrt{np(1 - p)}}$ and use the standard normal CDF.)

63 (2 points) Consider a European call option with strike $K = 44.153386$ and maturity $T = 1$. The price of the underlying asset is 42, and $r = 5\%$, $\delta = 0$. What is the futures price on the underlying asset with maturity $T = 1$? If the price of the call option is 6.6578, what is the price of a European put option with strike $K = 44.153386$ and maturity $T = 1$?

- The futures price is 44.153386. The put option price is 6.6578.
- The futures price is 42.153386. I don't have enough information to find the put option price.
- The futures price is 44.153386. The put option price is 3.3455.

- d. The futures price is 46.153386. I don't have enough information to find the put option price.

64 (2 points) A cash-or-nothing option pays 10 at time $T = 1$ only if the underlying price S_T is greater than the strike $K = 35$. The price of the cash-or-nothing option is

$$10e^{-rT}N(d_2)$$

A gap call option pays 0 at time $T = 1$ if $S_T < 35$ and $S_T - 45$ if $S_T > 35$. What position in a plain vanilla call with strike $K = 35$ and in the above cash-or-nothing option replicates the gap call option? What is the price of the gap call option? Assume $S = 42$, $\sigma = 0.3$, $r = 0.05$, $\delta = 0$, $T = 1$, and use $N(d_1) = 0.8227$ and $N(d_2) = 0.7340$.

- a. The gap call option can be replicated by long one vanilla call and short one cash-or-nothing option. The price of the gap call option is 3.13.
- b. The gap call option can be replicated by long one vanilla call and long one cash-or-nothing option. The price of the gap call option is 2.06.
- c. The gap call option can be replicated by long one vanilla call and short two cash-or-nothing options. The price of the gap call option is 3.13.
- d. The gap call option can be replicated by short one vanilla call and short one cash-or-nothing option. The price of the gap call option is 1.16.

65 (2 points) The time value of a European straddle is the difference between the value of the straddle and the intrinsic value $\max(S_T - K, 0) + \max(K - S_T, 0)$. Thus, the time value of a European straddle is highest when:

- a. $r > \delta$
- b. $K = S_0$
- c. K is the delta-symmetric strike
- d. K is the futures price with the same maturity as the straddle

66 (2 points) Consider two options, a European call and a European put, both written on the same underlying, with the same strike and the same maturity. The implied volatility of the call is higher than the implied volatility of the put. Is there an arbitrage opportunity? How would your answer change if the options had different strikes? How would your answer to the first question change if the options had different maturities?

- a. Yes, there is an arbitrage opportunity (through the put-call parity). There is no arbitrage opportunity if the options have different strikes, but there is an arbitrage opportunity if the options have different maturities.
- b. There is an arbitrage opportunity in all cases.

- c. Yes, there is an arbitrage opportunity (through the put-call parity). There is no arbitrage opportunity if the options have different strikes. There is no arbitrage opportunity if the options have different maturities.
- d. None of these cases represent an arbitrage opportunity.

67 (2 points) Stock XYZ is trading at \$43 and pays no dividends. If the six-month 45-strike European call and European put are equal in price, what is the six-month risk-free interest rate?

- a. I need the volatility in order to answer this question.
- b. 6.45%
- c. The call and put cannot be equal in price, since the asset does not pay dividends.
- d. 9.09%

68 (2 points) A stock is priced at \$40 with no dividend payments, and the risk-free interest rate is 2% (continuously compounded). The price of a call option with a strike of 35 is \$3.35 higher than the price of a call option with a strike of 40, and both options expire in 3 months. Find the difference between an otherwise equivalent 40-strike put option and an otherwise equivalent 35-strike put option.

- a. The difference between $P(40)$ and $P(35)$ is \$1.55.
- b. By the put-call parity, the difference between $P(40)$ and $P(35)$ is the same as the difference between $C(35)$ and $C(40)$, and equals \$3.35.
- c. Any difference between $P(40)$ and $P(35)$ would generate an arbitrage opportunity. Thus, $P(40)$ and $P(35)$ must have the same price.
- d. The difference between $P(40)$ and $P(35)$ is \$1.63.

69 (2 points) Consider a put option with strike 55 and maturity 4 months on a non-dividend-paying asset with spot price 55 and volatility of returns $\sigma = 0.3$. Assume that the risk-free rate is 4.5%.

What is the risk-neutral probability that the put will expire in the money? What is the delta of the put option?

(Hint: Find d_2 first to get the probability. Then, use the relationship $d_1 > d_2$ to find Δ_P by elimination.)

- a. The risk-neutral probability is 0.5. The delta is 0.5675.
- b. The risk-neutral probability is 0.5. The delta is -0.4325.
- c. The risk-neutral probability is 0.4325. The delta is -0.5.

d. The risk-neutral probability is 0.5. The delta is -0.5.

70 (2 points) Stock XYZ is currently priced at \$50. It pays no dividends. The one-year maturity 60-strike European call and put are trading at \$8 and \$11, respectively. What is the one-year forward price on the stock? What is the implied volatility on XYZ's stock returns?

(Hint: After finding the forward price, determine if you have enough information to find the implied volatility, and select the correct answer by elimination.)

- a. The forward price is given by \$57.69. The implied volatility is 46.29%.
- b. The forward price is given by \$57.69. The implied volatility is 12.29%.
- c. The forward price is given by \$56.60. I don't have enough information to compute the implied volatility.
- d. The forward price is given by \$56.60. The implied volatility is 46.29%.

71 (2 points) You short sell 3 call options of a non-dividend paying stock with a strike price of 35 and a premium per option of 4.13, while simultaneously buying 5 call options with a strike price of 40 and a premium per option of 2.78. Both options have a 3-month maturity. Assuming you hold your positions until maturity, what is the maximum profit and the maximum loss you can make on the entire option portfolio? The stock's current price is 40, and the risk-free interest rate is 5% continuously compounded.

- a. Maximum profit = Unlimited; Maximum loss = 16.53.
- b. Maximum profit = 16.53; Maximum loss = Unlimited.
- c. Maximum profit = 1.53; Maximum loss = Unlimited.
- d. Maximum profit = Unlimited; Maximum loss = 1.53.

72 (2 points) You are considering the purchase of a 3-month 41.5-strike American put option on a non-dividend-paying stock. The Black-Scholes framework holds. The stock is currently selling for 40, and the stock's return volatility is 30%. The current put option delta is -0.5. What is the risk-free rate? What is the current price of the option?

- a. The risk-free rate is 8.2256%. The option price is \$6.1854.
- b. The risk-free rate is 10.2256%. The option price is \$4.1854.
- c. The risk-free rate is 10.2256%. The option price is \$2.1854.
- d. Since this is an American option, we cannot apply the Black-Scholes formula to find the interest rate or the price of the option.

73 (2 points) Let V^+ denote the value of a bet that pays \$1 at time T if $S_T > K$:

$$V^+ = e^{-rT} N(d_2)$$

What is the value V^- of a bet that pays \$1 at time T if $S_T < K$? What is the strike price such that these two bets have equal value, i.e., $V^+ = V^-$?

- a. $V^- = e^{-rT}[1 - N(d_2)]$. The value of K for which $V^+ = V^-$ is $K = S_0 e^{(r-\delta+\sigma^2/2)T}$
- b. $V^- = e^{-rT}[1 - N(-d_2)]$. The value of K for which $V^+ = V^-$ is $K = S_0 e^{(r-\delta-\sigma^2/2)T}$
- c. $V^- = e^{-rT}[1 - N(-d_2)]$. The value of K for which $V^+ = V^-$ is $K = S_0 e^{(r-\delta+\sigma^2/2)T}$
- d. $V^- = e^{-rT}[1 - N(d_2)]$. The value of K for which $V^+ = V^-$ is $K = S_0 e^{(r-\delta-\sigma^2/2)T}$

74 (2 points) You want to create a delta-neutral calendar spread using European call options that are exactly at-the-money ($S_0 = K$). The spread consists of a long call with maturity T_L and a short call with maturity T_S , where $T_S < T_L$.

To make this spread delta-neutral, what must the short maturity T_S be?

- a. The maturity of the short option must be $T_S = T_L/2$.
- b. The maturity of the short option must be $T_S = 0$.
- c. It is impossible to create such a spread with $T_S < T_L$.
- d. This cannot be determined without knowing σ and r .

75 (2 points) What should be the strike for a 1-year put option to get a Delta of -0.4, with the underlying trading at 40? Let $\sigma = 0.3$, $r = 0.05$, and $\delta = 0$.

- a. To get a Delta of -0.4, we need to set the strike to 40.7671.
- b. To get a Delta of -0.4, we need to set the strike to 38.7287.
- c. To get a Delta of -0.4, we need to set the strike to 46.2221.
- d. I don't have enough information to answer this question.