UCLA Anderson School of Management
Daniel Andrei, Derivative Markets MGMTMFE 406, Winter 2018

# MFE – Final Exam

March 2018

**Date:** ______________________

**Your Name:** ______________________

**Your email address:** ______________________

**Your Signature:**[1] ______________________

- This exam is open book, open notes. You can use a calculator or a computer, **but be sure to show or explain your work**.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- If you are stuck on something, make an assumption, tell me what it is, and do the best you can. I give partial credit if you provide enough correct information.

**TIME LIMIT: 3 hours**

**TOTAL POINTS: 100**

[1]As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.
By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.

**1 Implied volatility** (8 points) Consider a European call option on the S&P 500 with a strike $K = 2,750$ whose maturity is in 3 months from now. The level of the S&P 500 today is 2,750. The continuously compounded risk-free rate and the dividend yield on the S&P 500 are both equal to 3%. The price of the call option is \$92.53.

a. (4 points) What is your best guess for the implied volatility of S&P 500, *without using the Excel add-in formula? You must show your calculations to receive credit.*

| Implied volatility: |
|---|
| |

b. (4 points) The price of an European call option with maturity 1 year but otherwise identical to the previous call option is \$163.43. Which of these two options is relatively more expensive? Why? *You are not allowed to use any Excel add-in formula to answer this question. You must show your calculations to receive credit.*

**1** *Answers:*

a. Use the simple implied vol formula from Brenner and Subrahmanyam (1998):

$$\sigma \approx \frac{C_0}{S_0} \frac{1}{0.398\sqrt{T}} = 0.1691 \tag{1}$$

b. Using the same formula, the implied vol of the second option is

$$\sigma \approx \frac{C_0}{S_0} \frac{1}{0.398\sqrt{T}} = 0.1493, \tag{2}$$

and thus the first option is relatively more expensive (has a higher implied vol).

**2 Log contract** (8 points) Consider a derivative asset that has the following payoff in one year from now ($T = 1$):

$$L_T = e^{rT} \ln S_T, \tag{3}$$

where $r$ is the continously compounded risk-free rate and $S_T$ is the price of the underlying in one year from now. The price of the underlying today is \$2.72. The continuously compounded risk-free rate and the dividend yield of the underlying are both equal to 3%. It can be shown that the price of this derivative asset today equals:

$$L_0 = \ln S_0 - \frac{1}{2}\sigma^2 T, \tag{4}$$

where $\sigma$ is the volatility of the underlying.

a. (4 points) The price of the derivative asset is $L_0 = \$0.90$. What is the volatility of the underlying asset?

| Volatility: |
| --- |
| |

b. (4 points) Assume that you are selling two of these derivative assets today. How many units of the underlying asset you will have to trade if you want to hedge this position? Will you buy or sell those units?

| Hedge with the underlying: |
| --- |
| |

**2** *Answers:*

a. The volatility of the underlying is

$$\sigma = \sqrt{\frac{2(\ln S_0 - L_0)}{T}} = 0.4486 \tag{5}$$

b. The total position is now:

$$-2 \ln S_0 + \sigma^2 T, \tag{6}$$

which has a delta of $-2/S_0$. In order to hedge, we need to buy $2/S_0$ units of the underlying, i.e., 0.7353 units.

**3 Cryptofinance** (20 points) Choose the (unique) correct answer:

a. (4 points) Two nearly identical documents will have their SHA-256 hashes that are

  (i) Nearly identical
  (ii) Completely different, in both content and length
  (iii) Completely different in content, but nearly identical in length
  (iv) Exactly identical

b. (4 points) The simplest and most secure way to prove to someone that you possess a copy of a certain digital document that they possess is to:

  (i) Mail the document to them
  (ii) Send the pdf of the document to them
  (iii) Show them the SHA-256 hash of the document
  (iv) Encrypt and then email the document to them

c. (4 points) The total number of bitcoins that will ever be mined and exist are:

  (i) Unlimited
  (ii) 100 million
  (iii) 21 million
  (iv) 17 million now but will grow at 1% per year until the year 2140

d. (4 points) SHA-256 hash expressed in hexadecimal number system has:

  (i) 256 character long string
  (ii) Depends of the size of the original message
  (iii) 64 character long string
  (iv) Varies in length

e. (4 points) Consider the following hexadecimal numbers: `A=abc`, `B=aac`, `C=fac`, and `D=0ff`. The order of these numbers from the lowest to the highest is:

  (i) `A,B,C,D`
  (ii) `D,A,B,C`
  (iii) `D,B,A,C`
  (iv) `A,C,B,D`

**3** *Answers:* a. (iii), b. (iii), c. (iii), d. (iii), e. (iii).

**4 Greeks** (20 points) Below is some information about two options. Suppose you buy one 45-strike call, one 45-strike put, and 0.121 shares. The continuously compounded risk-free rate is 3%. The stock price is \$40 and the volatility is 30%. *You are to answer this question using only the information provided here.*

| Strike = 45 | Call | Put |
|---|---|---|
| Price | 3.4042 | 7.4679 |
| Delta | 0.4330 | -0.5539 |
| Gamma | 0.0309 | 0.0309 |
| Vega | 0.1632 | 0.1632 |
| Theta | -0.0067 | -0.0044 |
| Rho | 0.1531 | -0.3259 |
| Elasticity | 5.0881 | -2.9667 |

a. (4 points) If the share price rises \$1 right after you enter the position, what is the approximate change in the value of the position? Why?

| Change in value of position |
|---|
| |

b. (4 points) How large a change in the stock price is a one standard deviation change over one day? (1 year = 365 days)

| Change in stock price |
|---|
| |

c. (4 points) Suppose that over the course of one day, the stock price does not move. Approximately what will be the profit on your position?

| Change in value |
|---|
| |

d. (4 points) Suppose that over the course of one day, the stock price moves significantly more than one standard deviation. Will you show a profit or loss on the position? Why?

| Gain or Loss? |
|---|
| |

e. (4 points) Suppose that the annual continuously compounded expected return on the stock is 10% and recall that the continuously compounded risk-free rate is 3%. What is the annualized expected return on the option and stock position? *Hint: Compute first the elasticity of the position, as a weighted average of individual component elasticities. Also, keep in mind that the elasticity of the stock is 1.*

| Expected Return |
|---|
| |

**4** *Answers:*

a. The delta of the position is

$$0.4330 - 0.5539 + 0.1210 = 0.0001$$

The change in value is approximately zero (you are almost delta-hedged).

b. Since $\sigma = 30\%$, the one-day change on a \$40 stock is approximately $\$40 \times 0.30 \times \sqrt{1/365} = \$0.6281$

c. If the share price does not move, there is time decay on the two options. Thus, the loss on the position when the stock price does not move is the sum of the thetas:

$$-0.0067 - 0.0044 = -\$0.0111$$

d. The gamma on the position is positive (both the purchased call and put have positive gammas), hence, a move in excess of the standard deviation will generate a profit.

e. The elasticity of the position is a weighted average of the individual component elasticities. Thus,

$$\text{Elasticity} = \frac{3.4042}{15.7121} \times 5.0881 + \frac{7.4679}{15.7121} \times (-2.9667) + \frac{0.121 \times 40}{15.7121} \times 1 = 0.0004$$

Thus, the expected return on the position is

$$r + \Omega(r_M - r) = .03 + 0.0004 \times (0.10 - 0.03) = 0.0300$$

The expected return on the (almost perfectly hedged) stock and option position is the risk-free rate.

**Assume the following for problems 5 and 6:**

- The stock price is \$100.
- The **effective** annual risk-free rate is 10%. This means that if you invest \$1, after one year you will have \$1.10.
- Here are option prices for you to use as necessary (these are Black-Scholes prices for options with one year to maturity):

| Strike | 80 | 90 | 100 | 110 | 120 |
|---|---|---|---|---|---|
| Calls | 27.675 | 19.675 | 12.993 | 7.966 | 4.555 |
| Delta Calls | 0.955 | 0.865 | 0.718 | 0.540 | 0.369 |
| Puts | 0.402 | 1.493 | 3.902 | 7.966 | 13.646 |

## 5 Synthetic forward, put-call parity (12 points)

a. (4 points) Using the interest rate and option information above, explain what option transaction you could use to create a synthetic long forward contract. Explain why this position is equivalent to a long forward contract.

b. (4 points) What is the dividend yield on this stock?

| Dividend yield: |
|---|
| |

c. (4 points) What are the deltas of the five put options above?

| Delta put(K=80): |
|---|

| Delta put(K=90): |
|---|

| Delta put(K=100): |
|---|

| Delta put(K=110): |
|---|

| Delta put(K=120): |
|---|

**5** *Answers:*

a. Buy the 110 strike call and sell the 110 strike put. This is equivalent to a forward because it requires no initial investment and you will buy the stock after one year: If $S > 110$ you exercise the call and if $S < 110$ the put is exercised.

b. According to the previous question, the forward price is \$110. Since the effective interest rate is 10%, the dividend yield must be 0.

c. The put-call parity on a non-dividend paying stock writes

$$C_0 - P_0 = S_0 - PV(K), \tag{7}$$

from which we get a relationship between deltas of options with the same maturity and the same strike,

$$\Delta_C - \Delta_P = 1, \tag{8}$$

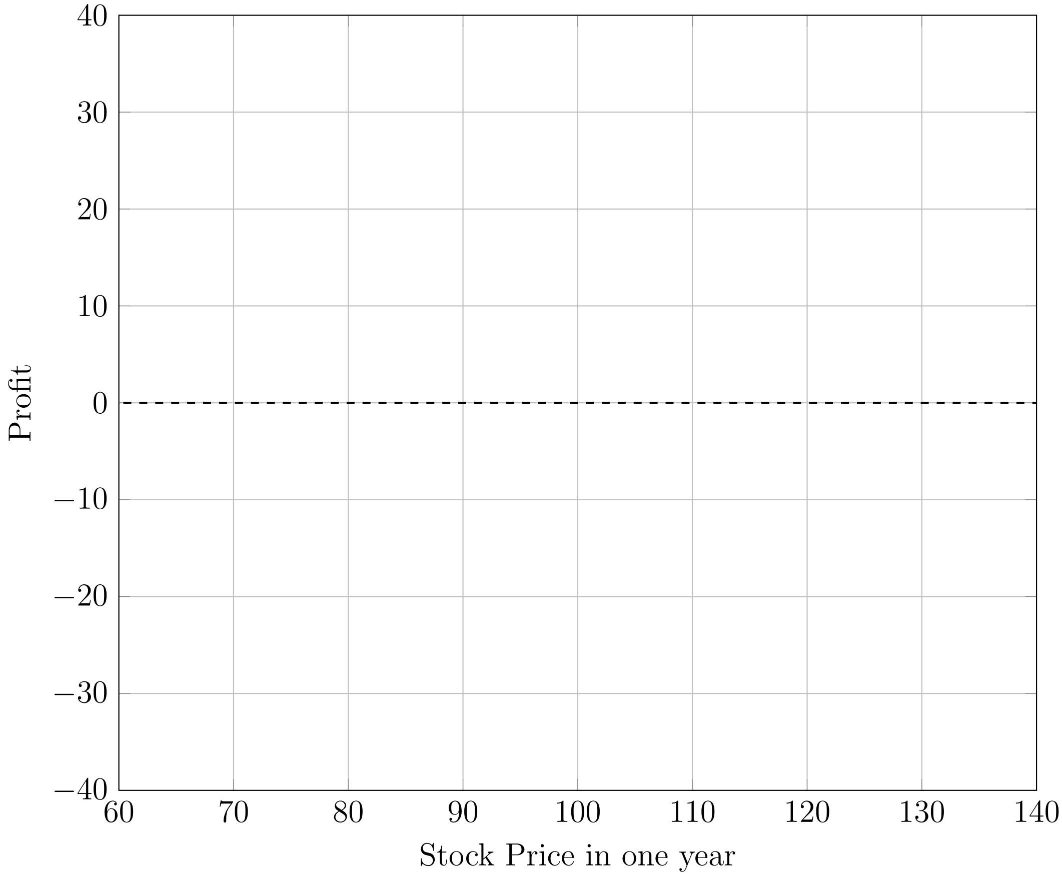
and thus

$$\Delta_P = \Delta_C - 1. \tag{9}$$

We obtain -0.045, -0.135, -0.282, -0.460, -0.631.

## 6 **Strangle** (12 points)

a. (4 points) Draw a **profit** diagram for the following position: buy one 90-strike call, buy one 110-strike put.




b. (4 points) What is the height of the profit graph at $S = \$100$?

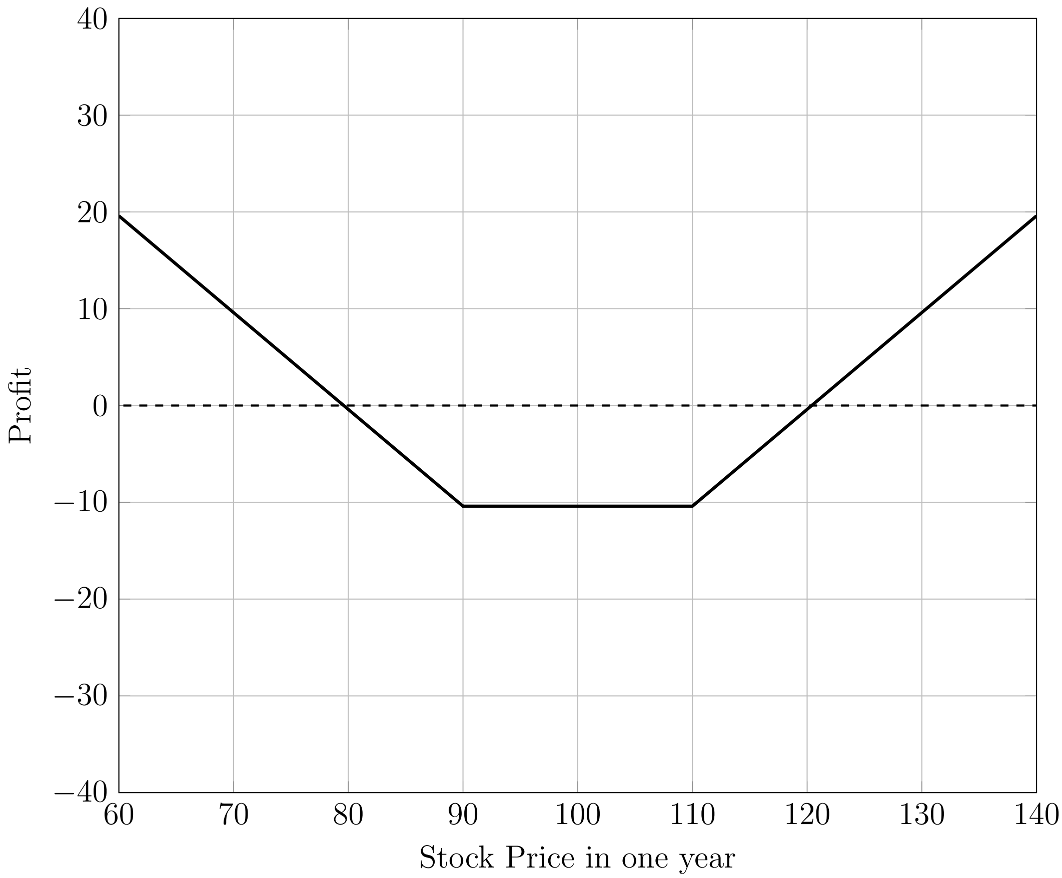
| Height at \$100: |
| --- |
| |

c. (4 points) What is the delta of this position?

| Delta strangle: |
| --- |
| |

**6** *Answers:*

a. The picture is identical to that of a strangle. The cost of the two options is \$27.64, with a future value of $27.64 \times 1.1 = \$30.40$.




b. The height of the profit graph at $S = \$100$? is

$$\$10 + \$10 - \$30.40 = -\$10.40 \tag{10}$$

c. The delta is $0.865 - 0.460 = 0.405$

**7 Black Scholes** (4 points) Consider a 3-month American call option with strike \$41.5 on a non dividend-paying stock. The current price of the stock is \$40 and its annualized volatility is 30%. Assume that the Black-Scholes framework holds and that the delta of the call option today is 0.45. *The continuously compounded risk-free rate is not given.*

Determine the current price of the American call option, $C_0$.

| Price of call option $C_0$: |
|---|
| |

**7** *Answer:*

It is never optimal to exercise an American call before maturity if the stock pays no dividends. Thus, we can price the call option using the Black-Scholes formula

$$C_0 = S_0 N(d_1) - Ke^{-rT} N(d_2) \tag{11}$$

We know that the delta of the option is $N(d_1)$ and in our case equals 0.45. Thus, $d_1 = -0.12566$. It follows that $d_2 = d_1 - \sigma\sqrt{T} = -0.27566$ and $N(d_2) = 0.39140$. We have

$$C_0 = 40 \times 0.45 - 41.5 \times e^{-r \times 0.25} \times 0.39140. \tag{12}$$

The only unknown here is $r$. Use the equation:

$$d_1 = \frac{\ln(40/41.5) + (r + 0.3^2/2)0.25}{0.3\sqrt{0.25}} \tag{13}$$

which gives $r = 0.02686$. Thus, $C_0 = \$1.8654$.

**8 Risk-Neutral Probabilities** (12 points) The current value of a stock is $S_0 = 85$. The stock pays dividends at a continuously-compounded rate of $\delta = 1\%$ and the continuously compounded risk free interest rate is $r = 4\%$. The volatility of the stock returns is $\sigma = 0.2$.

a. (4 points) What is the risk-neutral probability that the stock will end up above its median in one year from now?

| Risk-neutral probability: |
|---|
| |

b. (4 points) What is the median value of the stock in one year from now?

| Median: |
|---|
| |

c. (4 points) Do you think the mean value of the stock in one year from now (under the risk-neutral distribution) is located above or below the median? Justify your answer.

**8** *Answers:*

a. The risk-neutral probability that the stock will end up above its median in one year from now is

$$\mathbb{P}^Q\left[S_T > median\right] = 0.5. \tag{14}$$

b. The probability above is $N(d_2)$, and thus $d_2 = 0$, which yields:

$$\ln\left(\frac{S_0}{median}\right) + \left(r - \delta - \frac{\sigma^2}{2}\right) T = 0, \tag{15}$$

and thus the median is

$$median = S_0 e^{\left(r-\delta-\frac{\sigma^2}{2}\right)T} = 85.8543. \tag{16}$$

c. Due to the log-normality of the price (i.e. positive skewness) the mean is higher than the median.

**9 Risk-Neutral Density** (4 points) For a Black-Scholes **put option**, compute its second derivative with respect to its strike and show that this derivative is directly linked to the risk neutral density denoted $p(S_T, T \mid S_t, t)$. *You must show calculations to receive credit.*

**9** *Answer:*

The price of the Black-Scholes put option is:

$$P(K) = e^{-r(T-t)} \int_0^K (K - S_T) p(S_T, T \mid S_t, t) dS_T \tag{17}$$

$$\frac{\partial P}{\partial K} = e^{-r(T-t)} \int_0^K p(S_T, T \mid S_t, t) dS_T \tag{18}$$

$$\frac{\partial^2 P}{\partial K^2} = e^{-r(T-t)} p(K, T \mid S_t, t). \tag{19}$$

For the second and third equality, we have used Leibniz's rule for differentiation under the integral sign.