UCLA Anderson School of Management
Daniel Andrei, Derivative Markets MGMTMFE 406, Winter 2018

MFE – Midterm

February 2018

Date: ____________________

Your Name: ____________________

Your Signature:[1] ____________________

- This exam is open book, open notes. You can use a calculator or a computer, **but be sure to show or explain your work**.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- **If you are stuck on something, make an assumption, tell me what it is, and do the best you can.** I give partial credit if you provide enough correct information.

TIME LIMIT: 2 hours

TOTAL POINTS: 100

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.
By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help during the examination.

1 (24 points) Answer the following questions.

a. (4 points) The effective annual interest rate is 8%. What is the equivalent continuously compounded interest rate?

Interest rate

b. (4 points) A stock has an annual volatility of 24%. What is the 1-month volatility?

1-month volatility

c. (4 points) How many nodes are there in a non-recombining binomial tree with N time steps?

Number of nodes

d. (4 points) When do you want to be short a call option on IBM stock?

e. (4 points) Assume no dividends. The stock price is \$100. The riskless interest rate is 5% per annum. Consider a one-year European put option with strike \$104. If the volatility is zero, what is the put worth?

Price of put option

f. (4 points) How many nodes are there in a recombining binomial tree with N time steps?

Number of nodes

1 Solution

a. $\ln(1.08) = 0.077$.

b. The 1-month volatility is $0.24 \times \sqrt{1/12} = 0.0693$.

c. The number of nodes is

$$2^0 + 2^1 + 2^2 + ...2^N = 2^{N+1} - 1 \tag{1}$$

d. You want to be short a call if you expect the call to fall in price (e.g., the underlying is expected to fall or the volatility is expected to fall).

e. The required rate of return on the stock is the riskless interest rate. Thus, the option finishes out-of-the money:

$$100 \times e^{0.05} = 105.127 > 104, \tag{2}$$

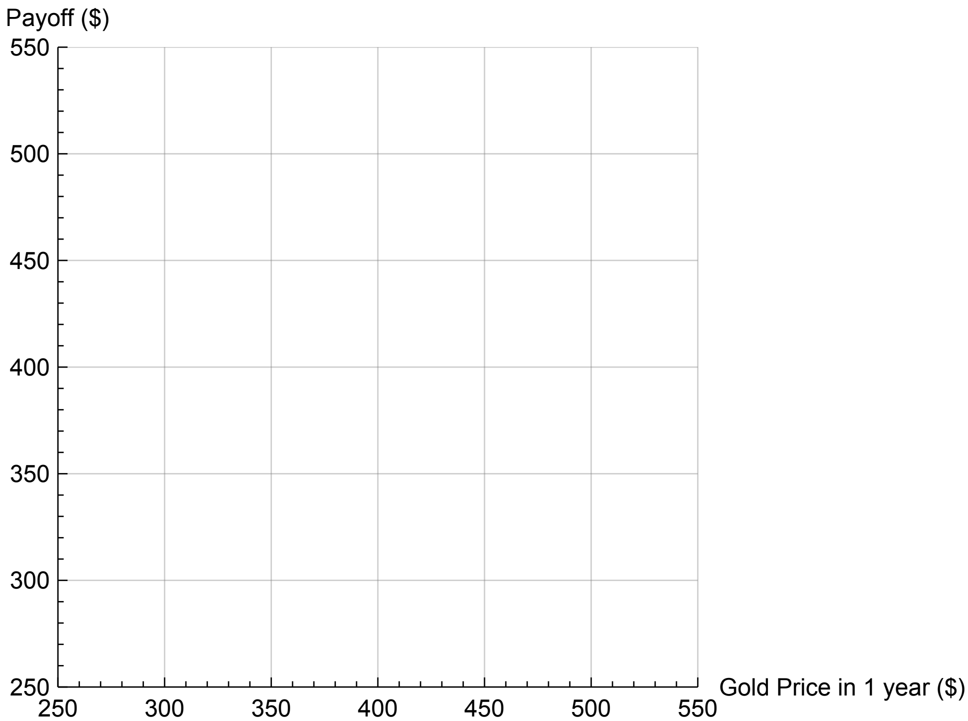
and is therefore worthless.

f. The number of nodes is

$$1 + 2 + ... + N + 1 = \frac{(N+1)(N+2)}{2} \tag{3}$$

2 Risk management (16 points) *Goldfield* is a gold-mining firm planning to sell 10,000 ounces of gold precisely 1 year from today. *Goldfield* hopes that the gold price will rise over the next year. Gold insurance (i.e., a put option) provides a way to have higher profit at high gold prices while still being protected against low prices. That is, the put option provides a **floor** on the price.

a. (4 points) Suppose that the market price for a 400-strike put option is \$6.775/oz. *Goldfield* decides to buy a put option for every ounce of gold it plans to sell. Draw the **payoff** diagram for this hedged position (i.e., the payoff resulting from selling one ounce of gold and exercising one 400-strike put option).



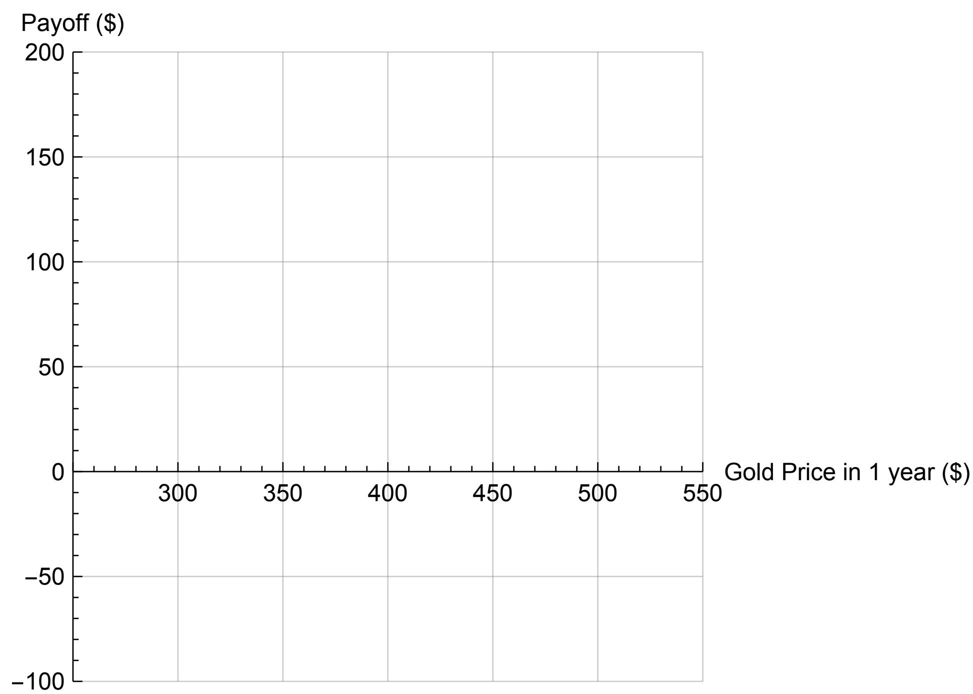
b. (4 points) A disadvantage to buying a put option is that *Goldfield* pays the premium even when the gold price is high and insurance was, after the fact, unnecessary. One way to avoid this problem is a **paylater** strategy, where the premium is paid only when the insurance is needed.

Consider the following strategy for *Goldfield*: Buy two 400-strike puts and sell a 450-strike put. The market price for a 450-strike put option is \$13.55/oz.

What is the net option premium of this strategy?

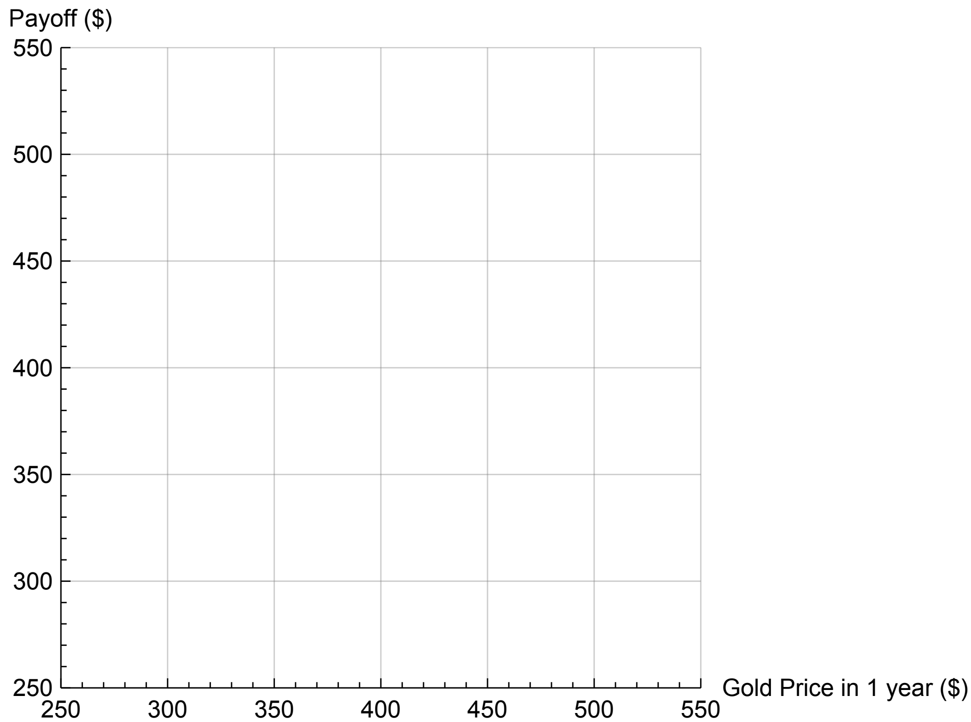
Net option premium

c. (4 points) Draw the **payoff** diagram for this option position (i.e., buy two 400-strike puts and sell a 450-strike put).



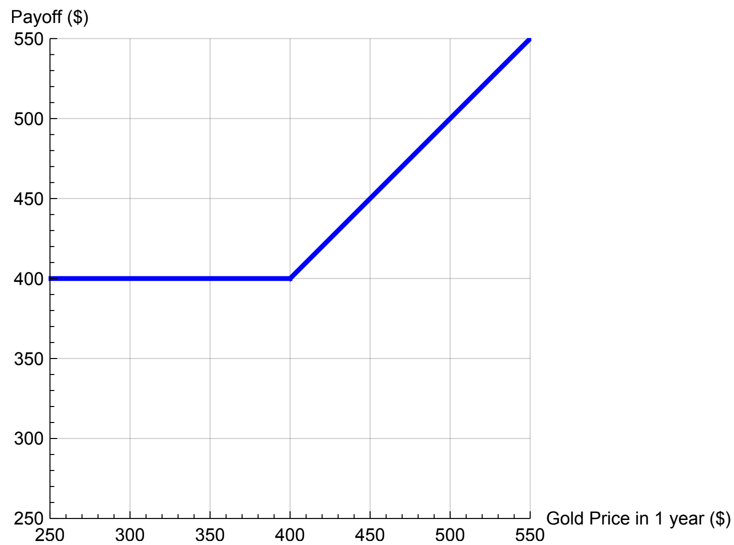
d. (4 points) Draw the **payoff** diagram for the **paylater** hedged position (i.e., the payoff resulting from selling one ounce of gold and entering in one **paylater** strategy).

What is the height of the plot at \$450? Clearly indicate this point on the diagram. Discuss the difference between this hedged position and the hedged position from point a.



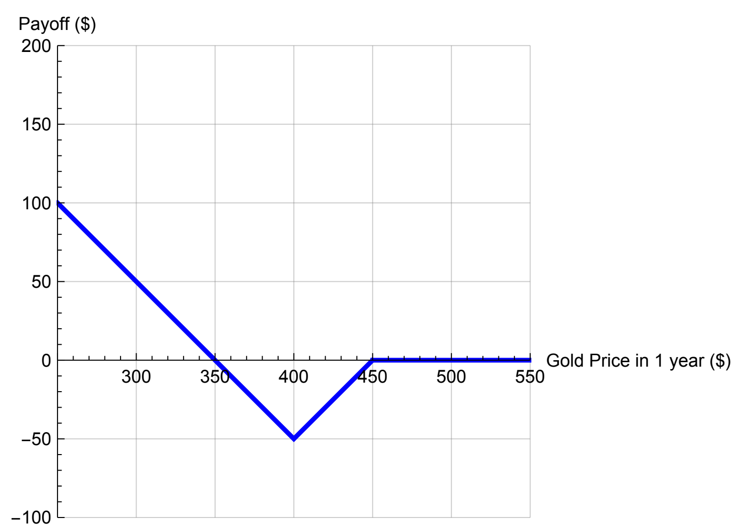
2 Solution

a. The payoff of the put hedged position is

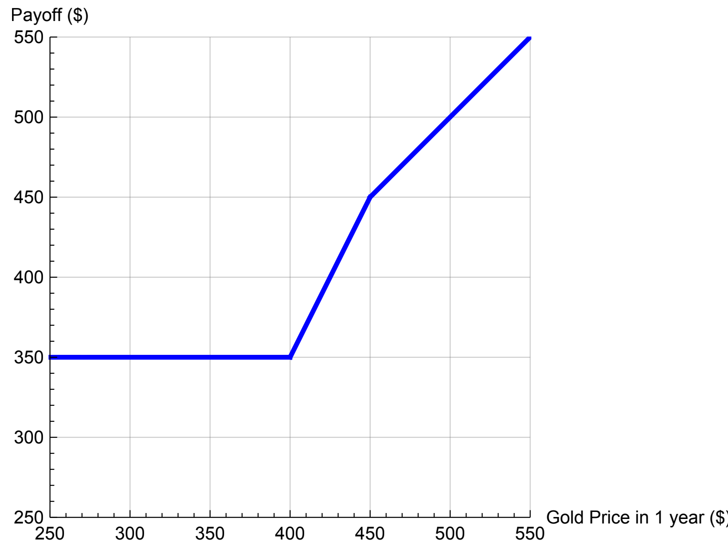


b. The net option premium is $2 \times \$6.775 - \$13.55 = \$0$.

c. The payoff of the option position is



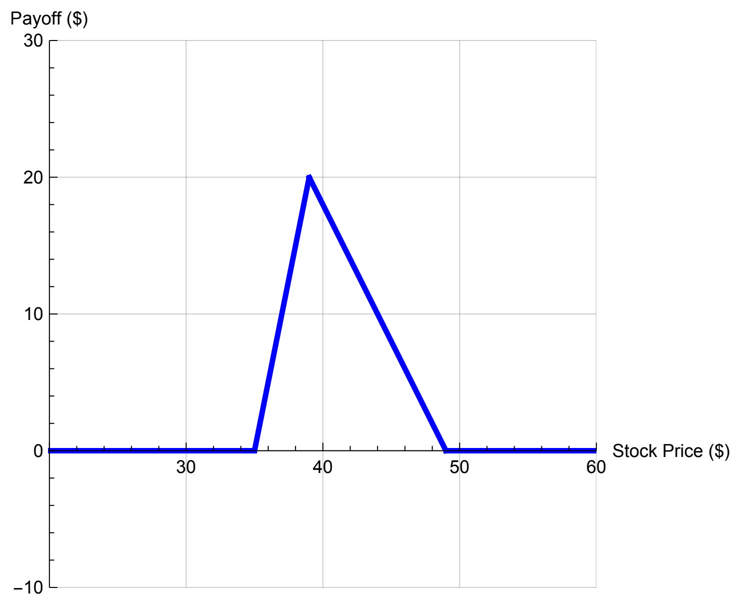
d. The payoff of the **paylater** hedged position is



When the price of gold is greater than \$450, neither put is exercised, and *Goldfield*'s payoff is the same as if it were unhedged. When the price of gold is between \$400 and \$450, because of the written 450-strike put, the firm loses \$2 of payoff of every \$1 decline in the price of gold. Below \$400, the purchased 400-strike puts are exercised, and the payoff becomes constant.

The net result is an insurance policy that is not paid for unless it is needed (in contrast with point a, where the put is paid even when insurance is unnecessary). However, when the gold price is below \$450, the paylater hedging strategy does worse because it offers less insurance.

3 Asymmetric Butterfly Spread (16 points) Below is a **payoff** diagram for a position. All options have 1 year to maturity and the stock price today is \$40. The underlying asset (the stock) is not paying any dividends.



a. (4 points) Can the portfolio corresponding to the above payoff have zero or negative initial premium? Justify your answer.

b. (4 points) The "kinks" of the payoff diagram are located at \$35, \$39, and \$49. Consider 3 options: A call option with strike \$35, a call option with strike \$39, and a call option with strike \$49. Using these options, find the quantities that construct the diagram.

Asset	Call(35)	Call(39)	Call(49)
Position			

c. (4 points) Consider 3 options: A call option with strike \$35, a put option with strike \$39, and a call option with strike \$49. Using these options **and the stock**, find the quantities that construct the diagram. *(Hint: Start from the more familiar situation of point b; then use the put-call parity.)*

Asset	Stock	Call(35)	Put(39)	Call(49)
Position				

d. (4 points) What should be the position in the risk-free asset in order to get **exactly** the payoff above? (assume an interest rate of 0%)

Position in the risk-free asset

3 Solution

a. No. That would imply existence of arbitrage.

b. The distance between 35 and 39 is 4, and the height of the diagram at 39 is 20. This means that the slope from 35 to 39 is:

$$Slope = \frac{20}{4} = 5 \tag{4}$$

We should therefore buy 5 calls with strike \$35.

The distance between 39 and 49 is 10. This means that the slope from 39 to 49 is:

$$Slope = -\frac{20}{10} = -2 \tag{5}$$

In order to obtain a slope of -2 from a slope of 5, we need to sell 7 calls with strike \$39.

Finally, in order to obtain a slope of 0 after 49, we need to buy 2 calls with strike \$49. The solution is then:

5 Call(35), -7 Call(39), 2 Call(49)

c. The put call parity for options struck at 39 writes:

$$Call(39) - Put(39) = Stock - PV(39) \tag{6}$$

Thus, we can replace $Call(39)$ from above in the position obtained at point b:

$$5Call(35) - 7Call(39) + 2Call(49) = 5Call(35) - 7\left[Put(39) + Stock - PV(39)\right] + 2Call(49) \tag{7}$$

Therefore, the solution is:

-7 Stocks, 5 Call(35), -7 Put(39), 2 Call(49)

d. The position in the risk-free asset resulting from the equation above is:

$$7PV(39) = 7 \times \$39 \tag{8}$$

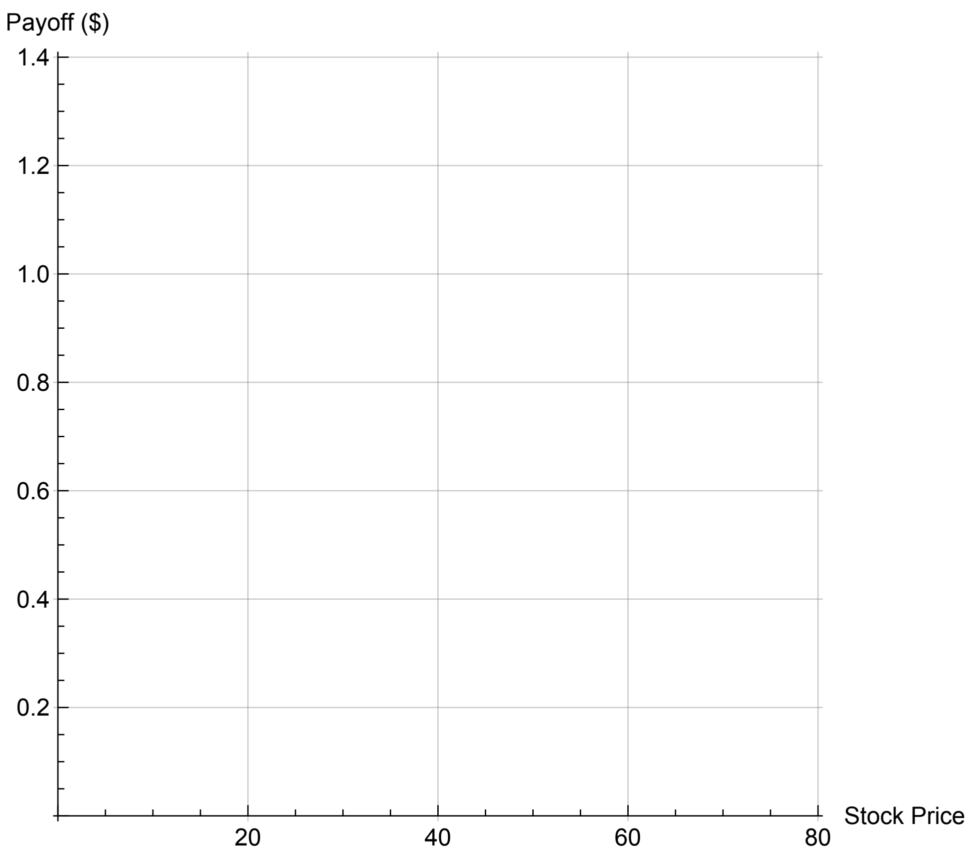
We therefore have to lend $7 \times \$39 = \$273$.

4 Log call option (28 points) A call option whose payoff is based on the log of the price of an asset is called **log call option**. Such an option has a nonlinear payoff at maturity. Its payoff is:

$$\max\left[\ln\left(\frac{S_T}{20}\right), 0\right] \tag{9}$$

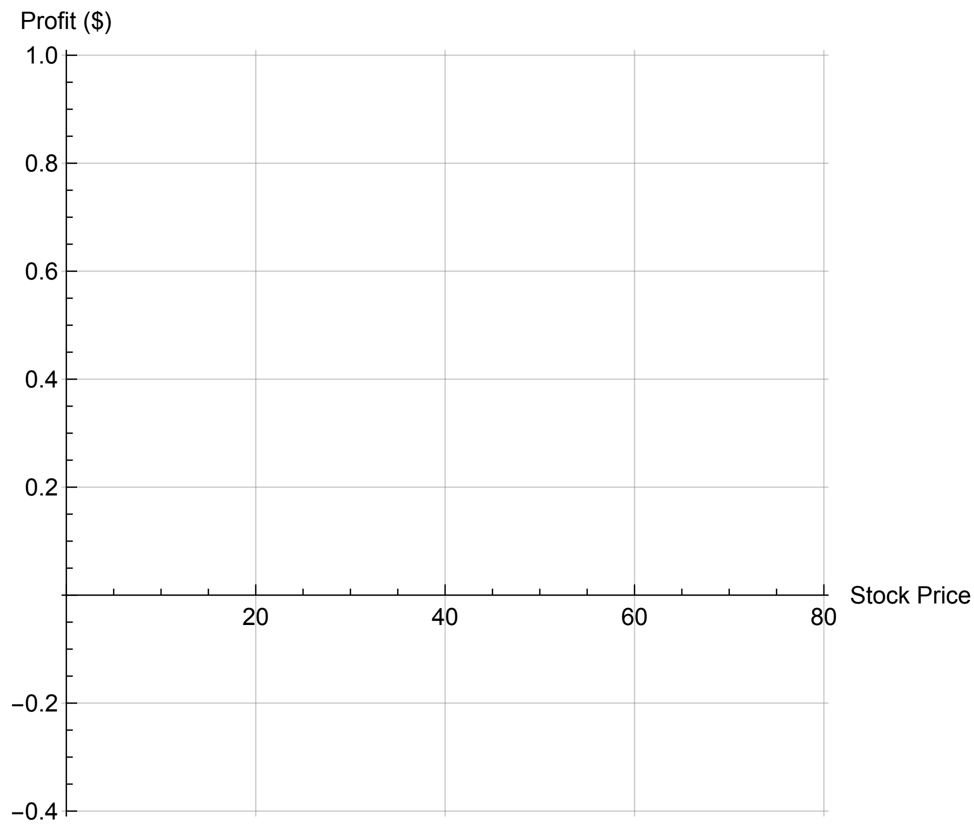
where the maturity T is 1 year. That is, the option pays the **log** of the stock price dividend by 20 if the owner of the option chooses to exercise it. For example, if the stock price is \$80, the claim when exercised pays $\ln(80/20) = \$1.39$.

a. (4 points) Draw the **payoff** diagram of the log call option. What is the height of the plot at \$40? What is the height of the plot at \$60? Clearly indicate these points on the diagram.



For the rest of this exercise, assume that the annual continuously compounded interest rate is $r = 15\%$, the annual dividend yield on the stock is $\delta = 10\%$, and the annual volatility of returns is 50%.

b. (4 points) The price of an European log call option today is \$0.348987. Draw the **profit** diagram of the log call option. What is the break-even point (i.e., the point where the profit of the position is zero)? Clearly indicate this point on the diagram.



The price of the stock today is $S_0 = \$30$. Consider a two-period binomial tree (i.e., there are 6 months between binomial nodes).

c. (4 points) Find u and d.

u and d:

d. (4 points) What is the risk-neutral probability of an up move?

Risk-neutral probability:

e. (8 points) At each node in the tree, fill in the prices for the American and European versions of this option (when you exercise the American version at time t, you receive $\max\left[\ln(S_t/20), 0\right]$, as at expiration). **Put an asterisk at each intermediary node where the American option is exercised.**

$S_0 = 30$
$Eur =$
$Amer =$

$S_1^u =$
$Eur =$
$Amer =$

$S_1^d =$
$Eur =$
$Amer =$

$S_2^{uu} =$
$Eur =$
$Amer =$

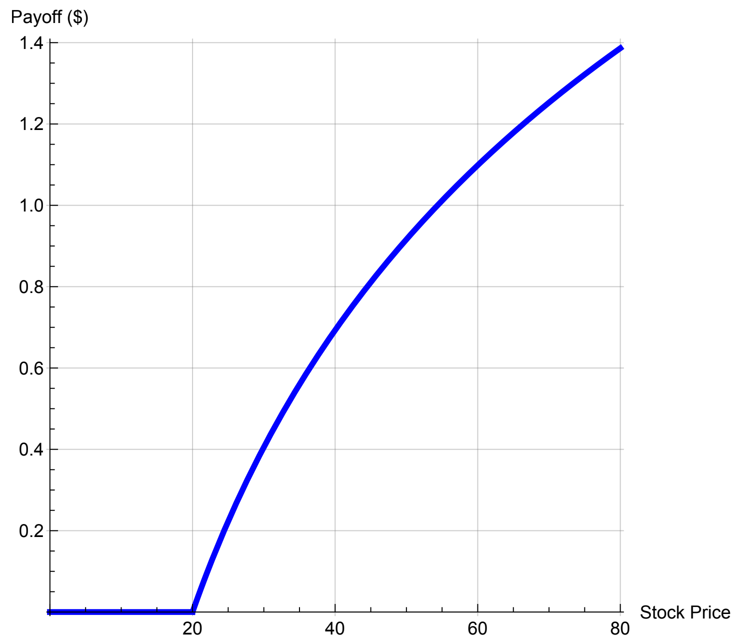
$S_2^{ud} =$
$Eur =$
$Amer =$

$S_2^{dd} =$
$Eur =$
$Amer =$

f. (4 points) Comparing the price of the European log call option that you found on the tree above with the market price of the option from point b, is there an arbitrage opportunity? If yes, does it involve buying or selling the market option?

4 Solution

a. The payoff of the **log call option** is

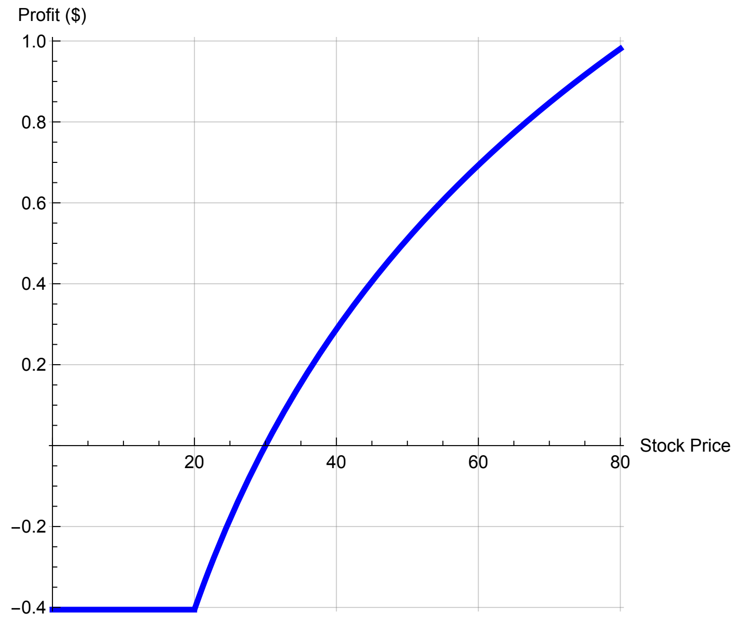


The height at $40 is $0.693. The height at $60 is $1.099.

b. The future value of the premium is:

$$FV(\$0.348987) = \$0.348987e^{0.15} = \$0.405465. \tag{10}$$

The profit diagram for the **log call option** is



The break-even point solves the equation

$$\ln\left(\frac{S_T}{20}\right) = \$0.405465, \tag{11}$$

that is, $S_T = 30$.

c. We obtain:

$$u = e^{(r-\delta)h+\sigma\sqrt{h}} = 1.4602 \tag{12}$$

$$d = e^{(r-\delta)h-\sigma\sqrt{h}} = 0.72 \tag{13}$$

d. The risk-neutral probability of an up move is

$$p^* = \frac{e^{(0.15-0.1)\times 0.5} - 0.72}{1.4602 - 0.72} = 0.4125$$

e. This claim is valued exactly like any other option. The value of the option at each node is given in the tree below.

$$
\begin{tikzcd}
 & & \begin{array}{r} S_2^{uu} = 63.9630 \\ Eur = 1.1626 \\ Amer = 1.1626 \end{array} \\
 & \begin{array}{r} S_1^{u} = 43.8051 \\ Eur = 0.6932 \\ Amer = 0.7840^{*} \end{array} \arrow[ur] \arrow[dr] & \\
\begin{array}{r} S_0 = 30 \\ Eur = 0.3603 \\ Amer = 0.4055^{*} \end{array} \arrow[ur] \arrow[dr] & & \begin{array}{r} S_2^{ud} = 31.5381 \\ Eur = 0.4555 \\ Amer = 0.4555 \end{array} \\
 & \begin{array}{r} S_1^{d} = 21.5989 \\ Eur = 0.1743 \\ Amer = 0.1743 \end{array} \arrow[ur] \arrow[dr] & \\
 & & \begin{array}{r} S_2^{dd} = 15.5505 \\ Eur = 0 \\ Amer = 0 \end{array}
\end{tikzcd}
$$

f. Yes, there is an arbitrage opportunity. The option is undervalued by the market ($\$0.348987 < \$0.3603$), so an arbitrage strategy would involve buying the option.

5 Arbitrage (8 points) The premium difference between otherwise identical European calls with different strike prices cannot be greater than the difference in prices. That is, if $K_1 < K_2$, then

$$C(K_1) - C(K_2) \leq K_2 - K_1 \tag{14}$$

Consider two European call options with the same maturity. The first option has a strike of \$60 and a premium of \$17. The second option has a strike of \$65 and a premium of \$11. Is there an arbitrage opportunity? If yes, describe the arbitrage strategy.

5 Solution

Yes, the inequality (14) is violated: the change in the option premium (\$6) exceeds the change in the strike price (\$5). The arbitrage strategy involves selling the low strike call and buying the high-strike call (a call bear spread), coupled with lending the proceeds at the risk-free rate:

Transaction	Time 0	$S_T < 60$	$60 \leq S_T \leq 65$	$65 < S_T$
Sell 60-strike call	17	0	$60 - S_T$	$60 - S_T$
Buy 65-strick call	-11	0	0	$S_T - 65$
Lend \$6	-6	FV(6)	FV(6)	FV(6)
TOTAL	0	$FV(6) > 0$	$60 + FV(6) - S_T > 0$	$FV(6) - 5 > 0$

where $FV(6)$ means "the future value of \$6."

6 Physical vs. risk-neutral probabilities (8 points) Consider a non-dividend paying stock whose price today is S_0 and in h years from now it can go up to S_u or down to S_d. The **physical** probability of the stock going up is denoted by p. The risk-free rate is r and the expected return of the stock is $\alpha > r$ (both are continuously compounded). The **risk-neutral** probability of the stock going up is denoted by p^*.

Find A in the following relationship between p^* and p:

$$p^* = p - A \tag{15}$$

Interpret this relationship.

6 Solution

We know that

$$S_0 = e^{-\alpha h}[pS_u + (1-p)S_d] \tag{16}$$

from which we obtain

$$p = \frac{e^{\alpha h} S_0 - S_d}{S_u - S_d} \tag{17}$$

Similarly

$$p^* = \frac{e^{rh} S_0 - S_d}{S_u - S_d} \tag{18}$$

Therefore, we can write

$$p^* = p - \frac{S_0 \left(e^{\alpha h} - e^{rh}\right)}{S_u - S_d} \tag{19}$$

This relationship shows that the actual probability, p, and the risk-neutral probability, p^*, differ by a term that is proportional to the dollar risk premium on the stock, $S_0 \left(e^{\alpha h} - e^{rh}\right)$. The actual probability, p, is reduced by the dollar risk premium per dollar of risk, $S_u - S_d$. Thus, the real and risk-neutral probabilities differ by an amount that is determined by investors' attitude toward risk.